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TIME INTEGRATION OF UNSTEADY TRANSONIC FLOW
TO A STEADY STATE SOLUTION BY
THE FINITE ELEMENT METHOD

by

Raymond John Nichols Jr.

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Thesis Advisor: D. J. Collins

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Time Integration of Unsteady Transonic Flow to a Steady State Solution by the Finite Element Method

by

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A secondary investigation applied the steady transonic small disturbance equations to a converging-diverging nozzle.

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I. INTRODUCTION

Transonic inviscid flows past a smooth airfoil may be expressed in terms of the velocity potential ϕ satisfying the transonic small disturbance equation,

$$(1 - M_\infty^2 - M_\infty^2(\gamma + 1)\phi_x)\phi_{xx} + \phi_{yy} = 0 \qquad (1)$$

This equation presents two major difficulties, 1) it is non-linear and 2) it is of the mixed hyperbolic-elliptic type. Analytical solutions to non-linear equations are difficult to obtain. One must normally resort to numerical methods. When the coefficient $(1 - M_\infty^2 - M_\infty^2 (\gamma + 1)\phi_x)$ in equation 1 is negative, the flow is supersonic and the equation is called hyperbolic; otherwise the flow is subsonic and the equation is elliptic. The forms of the two solutions are fundamentally different. Hyperbolic equations admit both discontinuities, which propagate only in characteristic directions, and the presence of shock fronts. Elliptic equations, on the other hand, require that the dependent variables and their derivatives be continuous and that a change in any part of the flow field affects every other part. Many non-linear elliptic equations are solved by appropriate relaxation iteration techniques by casting the equation in Poisson's form with the non-linearity acting as the driving force. Solutions to hyperbolic equations are usually obtained by the method of characteristics or by

finite difference marching techniques which use an artificial viscosity to represent the average jump conditions across the shock wave. In mixed supersonic and subsonic flows, normal numerical procedures break down because the boundary between the two regions is not known a priori.

Finite element numerical techniques have evolved as a powerful tool in obtaining approximate solutions to a wide variety of engineering problems, particularly ones with Neumann-Dirichlet boundary conditions, i.e., elliptical problems. They offer several outstanding advantages. Some of these are:

1) Non-homogeneous problems may be treated with relative simplicity.

2) Complex geometries may be modeled with relative ease since the elements can be graded in size and shape to follow boundaries of arbitrary shape.

3) Once the finite element model has been established, a variety of problems can be solved by supplying the computer with the appropriate data.

Chan and Brashears [Ref. 5] developed a finite element computer program for steady transonic flow over a non-lifting airfoil. This program uses the least squares method of weighted residuals to approximate equation 1 by a system of algebraic equations, and assembles the equations in a special way to account for the hyperbolic region of flow. This technique prevents the influence of downwind nodes from propagating upstream in the supersonic region.

The purpose of this thesis is to investigate the possibility of speeding the convergence of a solution by transforming the steady transonic equation to the unsteady equation

and integrating over time until the time dependent terms vanish and to extend the program of Ref. [5] to the transonic region of a converging-diverging nozzle.

II. DISCUSSION OF THE FINITE ELEMENT APPROACH

In a continuum problem of any dimension, the field variable, whether it is velocity potential, velocity, temperature, displacement or some other quantity, possesses infinitely many values because it is a function of each generic point in the solution region. Consequently, the problem is one of an infinite number of unknowns. The finite element approach subdivides the solution domain into a finite number of subdomains called elements and expresses the unknown field variable in terms of assumed approximating functions within each element. The approximating functions are sometimes called interpolating functions and are defined in terms of the values of the field variables at specified points called nodes or nodal points. Nodes usually lie on the element boundaries where adjacent elements are considered to be connected. In addition to boundary nodes, an element may also contain interior nodes. The nodal values of the field variable and the interpolating function for the elements completely define the bahavior of the field variable within the elements. Once these unknowns are found, the interpolating functions define the field variable throughout the assemblage of the element.

Clearly, the nature of the solution and the degree of accuracy of the approximation depends not only on the number and size of the elements used but also on the interpolating

functions selected. Interpolating functions may not be chosen arbitrarily because certain compatibility conditions must be satisfied. Often such functions are selected so that the field variable or its derivatives are continuous across adjoining element boundaries. Once the problem is formulated in terms of individual elements, the contributions of each element may be assembled to define the entire solution domain. This means, for example, that if we are treating a problem in stress analysis we can find the force-displacement or stiffness characteristics of each element and then assemble the elements to determine the stiffness of the whole structure. Finite element solutions are not, of course, restricted to structures problems, but the matrix of equations defined by the interpolating functions and the nodal field variables is still referred to as the stiffness matrix regardless of the field variable in the problem.

Solutions to continuous problems by the finite element approach always follow a systematic step-by-step process. This process is completely general to the finite element method and it is outlined below. [Ref. 4]

1. Discretize the continuum.

The first step is to divide the solution domain into elements. A variety of element shapes may be used and one or more different element shapes may be used in the same region. The type and number of the elements used in a given problem are a matter of engineering judgement.

2. Select the interpolating functions.

The next step is to choose the type of interpolating function to represent the variation of the field variable over the element. The field variable may be a scalar, a vector, or a higher order tensor. Often polynomials are selected as interpolating functions for they are easy to integrate and differentiate. The degree of polynomial chosen depends on the number of nodes and the nature and number of the unknowns and the continuity requirements imposed at the nodes and the element boundaries. The unknown quantities at the nodes may be assigned to the field variable and their derivatives.

3. Find the element properties.

After the elements and their interpolating functions have been selected, the matrix of algebraic equations which express the properties of the individual elements must be determined. Several methods are available for this task. These are: the direct approach, the variational approach, the method of weighted residuals and the energy balance approach. Reference [4] is a good source of information on the various techniques.

4. Assemble the element properties to obtain the system equations.

To find the properties of the over-all system, the matrix equations expressing the behavior of each element must be

added to the matrix equation of all other elements. The basis for this assembly procedure stems from the fact that connecting elements have common nodes and the field variable must be the same for each element sharing that node.

At this point the boundary conditions for the system of equations must be accounted for and the system of equations must be modified before it is ready for solution.

5. Solve the system of equations.

The assembly process of step 4 produces a set of simultaneous equations which can be solved to obtain the unknown field variables. Linear equations have a number of standard solution techniques readily available, but solutions to non-linear equations are more difficult to obtain.

6. Make additional computations if desired.

Important parameters, such as pressure coefficient in aerodynamic problems, may now be calculated from the values of the field variables.

III. THEORY AND BASIC EQUATIONS

A. STEADY TRANSONIC FLOW

Chan et al. [Ref. 5] developed an algorithm to analyze steady transonic flow over non-lifting thin airfoils. Boundary layer effects were ignored and the imbedded shock wave was assumed to be weak. These assumptions are consistent with transonic small disturbance theory which can be expressed mathematically by the following expressions.

$$(1 - M_\infty^2 - M_\infty^2(\gamma + 1)\phi_x)\phi_{xx} + \phi_{yy} = 0 \qquad (1)$$

Boundary conditions -

$$\nabla \cdot \phi = 0 \qquad \text{at infinity} \qquad (2)$$

$$v = (1 + u)dg/dx \qquad \text{on the airfoil} \qquad (3)$$

$$u = 0 \qquad \text{on the line of symmetry} \qquad (4)$$

where g(x,y) defines the airfoil and dg/dx describes the airfoil slope.

The above expressions appear in their dimensionless form where ϕ = perturbed velocity potential and the perturbed velocity components in the x and y directions are respectively defined as

$$u = \phi_x$$

$$v = \phi_y$$

M_∞ = freestream Mach number and γ = ratio of specific heats which for air is taken to be 1.4. The physical coordinates x' and y' and the velocity potential ϕ' are related to the dimensionless quantities by

$$x = x'/c, \qquad y = y'/c, \qquad \phi = \phi'/cU_\infty$$

where c is the chordlength of the airfoil and U_∞ is the free-stream velocity.

Once the flowfield solution is determined in terms of the perturbed velocities, the secondary unknowns are subsequently calculated. These include:

$$a = \left[\frac{\gamma-1}{2}(U_\infty^2 - V^2) + \left(\frac{U_\infty}{M_\infty}\right)^2\right]^{\frac{1}{2}} \tag{5}$$

$$M = \frac{V}{a} \tag{6}$$

$$\frac{P}{P_0} = \frac{1}{\left[1 + \frac{\gamma-1}{2}M^2\right]^{\gamma/(\gamma+1)}} \tag{7}$$

$$C_P = -\left[\frac{2u}{U_\infty} + (1-M_\infty^2)\frac{u^2}{U_\infty^2} + \frac{v^2}{U_\infty^2}\right] \simeq -\frac{2u}{U_\infty} \tag{8}$$

In the above, $U_\infty = 1$ is the normalized freestream velocity, a = local sound speed, p = local static pressure, M = local Mach number, V = total velocity, P_0 = stagnation pressure and C_p = pressure coefficient.

B. UNSTEADY TRANSONIC FLOW

Unsteady transonic inviscid flow may be expressed in terms of the velocity potential $\phi(x,y,t)$ to a first order approximation by

$$(1 - M_\infty^2 - M_\infty^2(\gamma + 1)\phi_x)\phi_{xx} + \phi_{yy} - 2M^2\phi_t - M^2\phi_{tt} = 0 \qquad (9)$$

which has the same non-linear coefficient retained for steady transonic flow in Equation 1.

Boundary conditions require that the disturbances vanish far from the airfoil,

$$\phi_x = 0 \qquad \phi_y = 0 \quad \text{at infinity} \qquad (10)$$

and that the flow remain attached to the body. Let $B(x,y,t) = 0$ define the body at any instant. The surface tangency restraint may now be expressed by the substantial derivitive DB/DT vanishing.

$$DB/DT = B_t + (1 + \phi_x)B_x + \phi_y B_y \qquad (11)$$

If the body remains stationary $B_t = 0$, and the tangency condition becomes the same as in steady flow, namely

$$v = (1 + u)\,dg/dx \qquad (12)$$

where dg/dx represents the airfoil slope.

The pressure coefficient for isentropic unsteady compressible flow is defined by

$$C_p = \frac{2}{\gamma M_\infty^2}\left\{[1 - \frac{\gamma - 1}{2} M_\infty^2 (2\phi_x + 2\phi_t + \phi_x^2 + \phi_y^2)]^{\gamma/(\gamma-1)} - 1\right\} \qquad (13)$$

Expanding by the binomial expansion and retaining only the first order terms gives,

$$C_p = -2\phi_x - 2\phi_t \qquad (14)$$

IV. FINITE ELEMENT FORMULATION

The method of weighted residuals is a technique for approximating solutions to linear or non-linear partial differential equations and it is the basis for the finite element formulation of the transonic small disturbance equation (Equation 1). This procedure involves assuming the general functional behavior of the field variable which would approximately satisfy the basic equation and boundary conditions. Substituting this approximation into the original differential equation results in some error called a residual. A system of algebraic equations results when a weighted average of the residual is forced to vanish as it is averaged over the entire domain.

A. STEADY FLOW

The approximate solution to equation 1 is assumed to be

$$\hat{\phi} = N_i \phi_i \tag{15}$$

where N_i are the interpolating functions which exhibits the behavior of equation 1 and ϕ_i are the undetermined parameters at the nodal points.

When $\hat{\phi}$ is substituted into equation 1, the resulting residual is

$$R = [1 - M_\infty^2 - M_\infty^2(\gamma + 1)N_{k,x}\phi_k]N_{j,xx} + N_{j,yy} \tag{16}$$

The weighted average is determined by multiplying the residual R by m linearly independent weighting functions W_i and integrating over the elemental domain. Forcing this residual to vanish yields,

$$\iint W_i R dA = 0 \qquad i = 1 \text{ to } m$$

Chan et al. [Ref. 5] found that when the weighting function W_i for equation 1 is chosen to be $\partial R/\partial \phi_i$ the resulting matrix is positive definite and well conditioned. This choice of weighting functions is referred to as the method of least squares because it is equivalent to minimizing the square of the residuals summed over the domain with respect to the undetermined parameters. That is,

$$\chi = \iint R^2 dA$$

$$\partial \chi/\partial \phi_i = \iint \partial R/\partial \phi_i R dA = 0 \tag{17}$$

Integrating over the domain produces the system of algebraic equations

$$S_{ij}\phi_j = 0 \tag{18}$$

where the elemental matrix S_{ij} is defined as

$$S_{ij} = \iint Q_j P_i \, dA \tag{19}$$

With Q_j and P_i equal to

$$Q_j = [1 - M_\infty^2 - M_\infty^2(\gamma + 1)N_{k,x}\phi_k] \; N_{j,xx} + N_{j,yy}$$

$$P_i = Q_i - M_\infty^2(1 + \gamma)N_{k,xx}\phi_k N_{i,x}$$

B. UNSTEADY FLOW

Development of the unsteady flow finite element equations is similar to the procedure used to formulate the finite element equations for steady transonic flow. The least squares method of weighted residuals is again used but ϕ is now a function of time as well as the spatial coordinates x and y.

The approximate solution has the form,

$$\hat{\phi} = N_i(x,y)\phi_i(t) \tag{20}$$

Substituting $\hat{\phi}$ in the unsteady transonic small disturbance equation, the weighted residual becomes,

$$\chi = \iint(R_1 + R_2 + R_3)^2 dA \tag{21}$$

where

$$R_1 = \{[1 - M_\infty^2 - M_\infty^2(\gamma + 1)\phi_k N_{k,x}]N_{j,xx} + N_{j,yy}\}\phi_j$$

$$R_2 = -2M^2 N_{j,x}\dot{\phi}_j$$

$$R_3 = -M^2 N_j \ddot{\phi}_j$$

Expanding equation 21 yields

$$\chi = \iint[R_1^2 + R_2^2 + R_3^2 + 2R_1R_2 + 2R_2R_3]dA \tag{22}$$

and minimizing with respect to the undetermined parameters ϕ_i the following system of algebraic equations result,

$$\phi_j = 0 = 2\iint \partial R_i/\partial\phi_j [R_1 + R_2 + R_3]dA$$

$$\partial R_1/\partial\phi_j = P_i$$

where P_i has been previously defined in the steady finite element formulation

The above equation may be rewritten in the form

$$S_{ij}\phi_j + SC_{ij}\dot{\phi}_j + SM_{ij}\ddot{\phi}_j = 0 \qquad (23)$$

The stiffness matrix S_{ij} is the same as that developed for the steady transonic equation and the damping (SC_{ij}) and mass (SM_{ij}) matrices are defined below.

$$SC_{ij} = -\iint M_\infty^2 N_{j,x} P_i \, dA \qquad (24)$$

$$SM_{ij} = -\iint M_\infty^2 N_j P_i \, dA \qquad (25)$$

V. ELEMENT DESCRIPTION AND ASSEMBLY OF EQUATIONS

A. ELEMENT DESCRIPTION

The basic element used in the finite element program is the non-conforming cubic triangular element developed by Bazeley et al. [Ref. 2]. Also used in the program are the quadrilateral and trapezoidal elements constructed from these triangular elements. These three types of elements can be mixed and used freely in the entire flow region except that only trapezoids should be used to cover the supersonic and mixed region in order to enact the special assembly procedures required by the hyperbolic equation which describes the flow in that region.

The basic triangular element is shown in Fig. 1, which at each vertex has the velocity potential and the velocity components as the undetermined parameters. This type of element was chosen because both Dirichlet and Neumann boundary conditions can be imposed with equal convenience. In addition, because velocity components are used as primary unknowns secondary parameters, such as Mach number and pressure coefficient can be calculated directly without resorting to numerical differentiation, which would produce additional errors.

In the element, the approximate solution is assumed as

$$\hat{\phi} = N_i \phi_i \qquad (i = 1 \text{ to } 9)$$

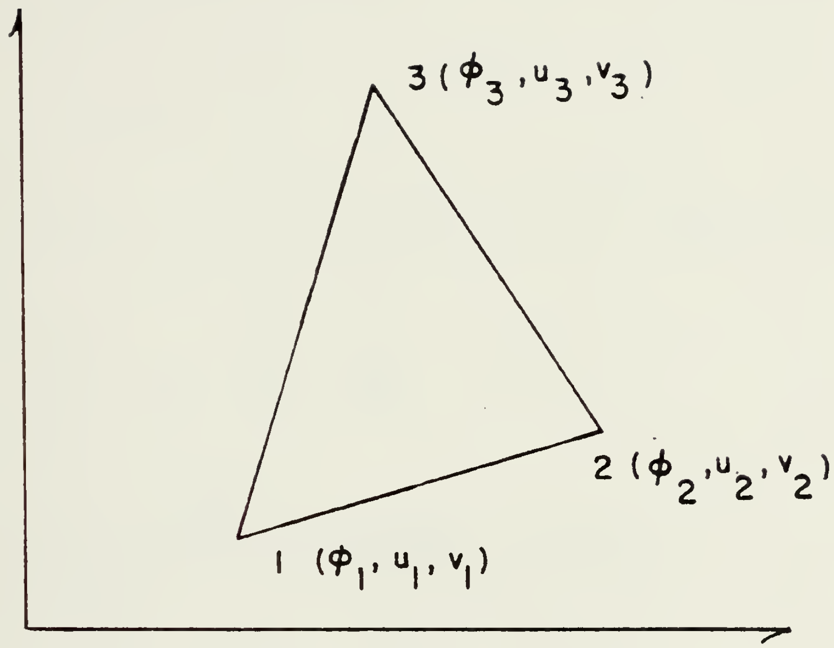


Figure 1 - Triangular Element

In which ϕ_i's are the nine undetermined parameters of ϕ and N_i are the interpolation functions which are expressed in terms of the area coordinates.

The shape or interpolating functions and their first and second derivitives are defined below.

Letting,

$$a_i = x_j y_k - x_k y_j$$

$$b_i = y_j - y_k$$

$$c_i = x_k - x_j$$

$$\Delta = \text{area of triangle 1-2-3} = (b_j c_k - b_k c_j)/2$$

$$\alpha = 0.5\ (c_k - c_j)$$

$$\beta = 0.5\ (b_j - b_k)$$

$$H = \zeta_k \zeta_j \zeta_k$$

$$H_x = b_i \zeta_j \zeta_k + b_j \zeta_k \zeta_i + b_k \zeta_i \zeta_j$$

$$H_y = c_i \zeta_j \zeta_k + c_j \zeta_k \zeta_i + c_k \zeta_i \zeta_j$$

$$H_{xx} = 2(\zeta_k b_j b_k + \zeta_j b_k b_k + \zeta_k b_i b_j)$$

$$H_{yy} = 2(\zeta_i c_j c_k + \zeta_j c_k c_i + \zeta_k c_i c_j)$$

with $i = (1,2,3), k = (3,1,2)$ then one has for $1 = (1,4,7)$, $i = (1,2,3)$

$$N_1 = \zeta_i^2 (3 - 2\zeta_i) + 2H$$

$$N_{1,x} = [6 b_i \zeta_i\ (1 - \zeta_i) + 2H_x]/2\Delta$$

$$N_{1,y} = [6 c_i \zeta_i\ (1 - \zeta_i) + 2H_y]/2\Delta$$

$$N_{l,xx} = [6b_i^2\ (1 - 2\zeta_i) + 2H_{xx}]\ /(2\Delta)^2$$

$$N_{l,yy} = [6c_i^2\ (1 - 2\zeta_i) + 2H_{yy}]\ /(2\Delta)^2$$

for l = (2,5,8), l = (1,2,3)

$$N_l = \zeta_i^2\ (c_k\zeta_j - c_j\zeta_k) + \alpha H$$

$$N_{l,x} = [2b_i\zeta_i(c_k\zeta_j - c_j\ {}_k) + 2\zeta_i^2 + \alpha H_x]\ /2\Delta$$

$$N_{l,y} = [2c_i\zeta_i(c_k\zeta_j - c_j\zeta_k) + \alpha H_y]\ /2\ \Delta$$

$$N_{l,xx} = [2b_i^2\ (c_k\zeta_j - c_j\zeta_k) + 4b_i(2\Delta)\zeta_i + \alpha H_{xx}]\ /(2\Delta)^2$$

$$N_{l,yy} = [2c_i^2\ (c_k\zeta_j - c_j\zeta_k) + \alpha H_{yy}]\ /(2\Delta)^2$$

for l = (3,6,9), l = (1,2,3)

$$N_l = \zeta_i^2\ (b_j\zeta_k - b_k\zeta_j) + \beta H$$

$$N_{l,x} = [2b_i\zeta_i(b_j\zeta_k - b_k\zeta_j) + \beta H_x]\ /2\Delta$$

$$N_{l,y} = [2c_i\zeta_i\ (b_j\zeta_k - b_k\zeta_j) + 2\zeta_i^2 + \beta H_y]\ /2\Delta$$

$$N_{l,xx} = [2b_i^2\ (b_j\zeta_k - b_k\zeta_j) + \beta H_{xx}]\ /(2\Delta)^2$$

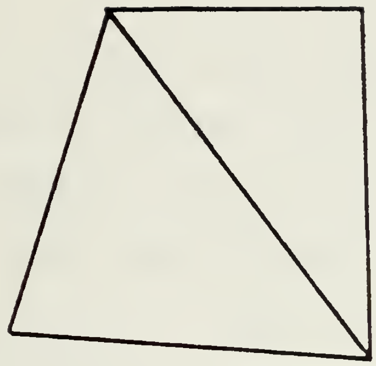
$$N_{l,yy} = [2c_i^2\ (b_j\zeta_k - b_k\zeta_j) + 4c_i(2\Delta)_i + \beta H_{yy}]\ /(2\Delta)^2.$$

Quadrilateral and trapezoidal elements as shown in Fig. 2 are also used in the present program, the former is used in the subsonic region and the latter in the mixed and supersonic region. The element matrix for the quadrilateral element is obtained by combining appropriately the matrices for two triangles, while the matrix for trapezoidal element is obtained by averaging contributions from two left-running and two right-running triangles. The averaging process is designed to remove the bias effects inherent in the quadrilaterals used.

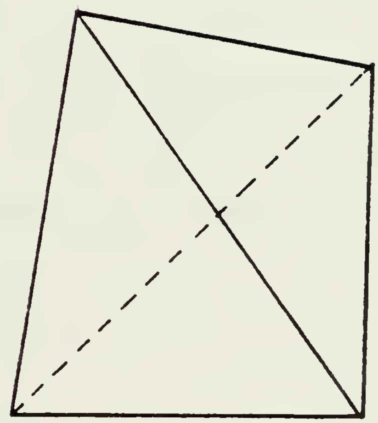
B. ASSEMBLY OF EQUATIONS

Straightforward application of the finite element assembly technique to transonic flows would fail (the solution diverges) because this would allow disturbances to propagate upwind in the supersonic region of flow where the governing equation is hyperbolic. Hyperbolic equations have a time-like dependency in that the solution at the downwind station is affected by the upwind station but not vice-versa. Assembly techniques for a transonic flow finite element program must take into account this time-like dependency. If the x-axis is taken as a time-like direction in the supersonic region, the element matrix may be assembled in a way similar to a backward finite difference operator, which has been successful in solving hyperbolic equations.

Consider the rectangular element sketched below with the upwind station I and the downwind station II, each having two nodal points with the element type chosen. The element matrices can be constructed in the usual manner.



a. Quadrilateral
Element



b. Trapezoidal
Element

Figure 2 - Quadrilateral and Trapezoidal Elements

However, before assembling the element matrix into the system matrix the non-linear coefficient of equation 1 is evaluated.

$$C = 1 - M_\infty^2 - M_\infty^2 (\gamma + 1) u$$

The sign of the coefficient being positive, zero, or negative defines the equation as elliptic, parabolic, or hyperbolic. If C is non-positive for all nodes in the element, the rows representing the improper downwind influence on the solution at an upwind station are ignored during assembly. This feature is automatically applied in the program requiring only a little care in arrangement of the nodes of the element. In the anticipated supersonic region, element node points should be arranged in the order as indicated in figure 3, starting with the upper left corner and proceeding in the counter-clockwise direction. In the elliptic region, i.e., where the coefficient is positive, no special assembly technique is invoked.

C. ITERATIVE PROCEDURES

With the equations assembled and the proper boundary conditions imposed, the system of non-linear algebraic equations is solved by iterative procedures in the form

$$S_{ij}(\tilde{\phi})\phi_j^{(n)} = 1_i \qquad (23)$$

to solve for the solution in the n^{th} iteration. The function $\tilde{\phi}$ is defined as

$$\tilde{\phi} = \theta\phi^{(n-1)} + (1-\theta)\phi^{(n-1)}$$

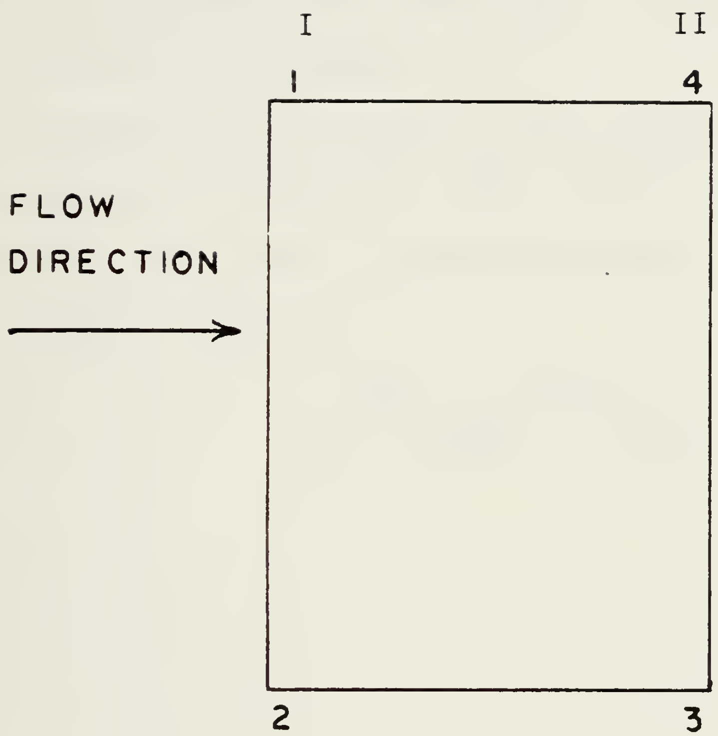


Figure 3 - Nodal Arrangement for Supersonic Region

in which the under-relaxation factor θ is in the range $0 < \theta \leq 1$. For subsonic flow $\theta = 1$, which is simply a successive approximation, yields good results, but it is necessary to under-relax somewhat with θ approximately .5 for supercritical flow. Generally, a smaller relaxation factor will make the solution more stable but it will tend to slow down the rate of convergence.

Equation 23 is subject to the convergence criterion that the change in local Mach number between two successive iterations is less than a prescribed value ε at all nodes in the flow field. That is,

$$\left| \frac{M^{(n)} - M^{(n-1)}}{M^{(n)}} \right| \leq \varepsilon .$$

VI. INTEGRATION OF UNSTEADY FINITE ELEMENT EQUATIONS

The unsteady transonic small disturbance equation (equation 9) when suitably reduced to a finite element approximation appears in the form,

$$S_{ij}\phi_j + SC_{ij}\dot{\phi}_j + SM_{ij}\ddot{\phi}_j = R_j \tag{26}$$

This equation is analogous to a damped spring mass system, hence S_{ij}, SC_{ij}, and SM_{ij} are respectively referred to as the stiffness, damping, and mass matrices.

Mathematically, equation 26 represents a system of second order differential equations with constant coefficients, which can be solved by standard numerical procedures for differential equations, such as Runge-Kutta or Milne methods. However, this would be a very costly technique if the coefficient matrices are very large. In practical finite element analysis there are a few effective methods which take advantage of the banded matrices usually encountered in finite elements. One such method is the direct numerical integration method.

Direct integration involves a numerical step-by-step procedure aimed at satisfying equation 26 only at discrete time intervals Δt apart and not over all time t. Conceptually, direct integration is a finite element method in space and a finite difference method in time. Examples of direct integration are the central difference method, Houbolt integration and the Wilson method. The first two schemes are finite

difference schemes whereas the latter is a linear acceleration method. Linear acceleration integration assumes a linear variation of acceleration from time t to time $t + \Delta t$.

Central differencing can be very effective in the solution of many dynamic problems especially those that involve a large system of equations. However, this method is unstable for all time steps larger than a critical time step.

Houbolt integration is an implicit finite differencing method related to central differencing, only it has the advantage of being stable for all TIME STEPS.

The Houbolt method was used to integrate the unsteady finite element equations because of this stability.

Houbolt integration uses the following finite difference expansions:

$$\phi_{i,t+\Delta t} = [2\phi_{i,t+\Delta t} - 5\phi_{i,t} + 4\phi_{i,t-\Delta t} - \phi_{i,t-2\Delta t}]/\Delta t^2$$

$$\phi_{i,t+\Delta t} = [11\phi_{t+\Delta t} - 18\phi_{i,t} + 9\phi_{i,t-\Delta t}\ 2\phi_{i,t-2\Delta t}]/6\Delta t$$

which are two backward-difference formulas with errors of order $(\Delta t)^2$.

The solution of $\phi_{i,t+\Delta t}$ must satisfy equation 26 and at time $t+\Delta t$ equation 26 becomes

$$S_{ij}\phi_{j,t+\Delta t} + SC_{ij}\dot{\phi}_{j,t+\Delta t} + SM_{ij}\ddot{\phi}_{j,t+\Delta t} = 0$$

Substituting the finite difference formulas for $\phi_{j,t+\Delta t}$ and rearranging all known vectors on the right hand side, the solution for $\phi_{j,t+\Delta t}$ is obtained, namely:

$$(a_0 SM_{ij} + a_1 SC_{ij} + S_{ij})\phi_{j,t+\Delta t} = R_{j,t+\Delta t} \tag{27}$$

$$+ (a_2 SM_{ij} + a_3 SC_{ij})\phi_{j,t} + (a_4 SM_{ij} + a_5 SC_{ij})\phi_{j,t-2\Delta t}$$

Where the constant integration coefficients are:

$$a_o = 2/\Delta t^2$$

$$a_1 = 11/6\Delta t$$

$$a_2 = 5/\Delta t$$

$$a_3 = 3/\Delta t$$

$$a_4 = 2a_0$$

$$a_5 = -a_3/2$$

$$a_6 = a_0/2$$

$$a_7 = a_3/9$$

Equation 27 may be written as

$$SE_{ij}\phi_{j,t+\Delta t} = RE_j \tag{28}$$

where the effective stiffness matrix SE_{ij} and the effective load vector RE_j are defined as:

$$SE_{ij} = S_{ij} + a_0 SM_{ij} + a_1 SC_{ij}$$

$$RE_j = SM_{ij}(a_2\phi_{j,t} + a_4\phi_{j,t-\Delta t} + a_6\phi_{j,t-2\Delta t})$$

$$+ SC_{ij}(a_3\phi_{j,t} + a_5\phi_{j,t-\Delta t} + a_7\phi_{j,t-2\Delta t})$$

Accurate knowledge of the vectors $\phi_{j,t-\Delta t}$ and $\phi_{j,t-2\Delta t}$ are required to yield an accurate solution for $\phi_{j,t+\Delta t}$ and

normally the Houbolt integration scheme requires a special starting procedure to determine the initial two vectors $\phi_{j,\Delta t}$ and $\phi_{j,2\Delta t}$. However, since the primary interest of this problem is to integrate the equations until they converge to a steady state solution, it is not necessary to obtain an accurate time history of the flow. Errors induced by the inaccurate starting vectors will vanish as time approaches infinity. Therefore, the starting vectors may be chosen somewhat arbitrarily.

VII. CONVERGING-DIVERGING NOZZLE

S. F. Shen [Ref. 10] demonstrated the feasibility of calculating compressible flows through a converging-diverging (Laval) nozzle by dividing the region of calculations into three patches, a subsonic region, a supersonic region and a transonic one, of course bounded by the other two regions. The locations of the boundaries for each region were chosen arbitrarily provided the sonic line is bracketed by the subsonic and supersonic boundaries.

Two different finite element formulations were used for the subsonic and supersonic regions, but Shen [10] resorted to analytical approximations to cover the transonic patch. This restricted the calculations to nozzles with small throat curvatures because no analytic solutions exist for nozzles with large throat curvatures. It is conceivable that STRANL-II could be adapted to provide a continuous solution throughout all three regions.

Outside the transonic region of flow the governing small disturbance equation is

$$(1 - M_\infty^2)\phi_{xx} + \phi_{yy} = 0 \qquad (29)$$

This holds for both subsonic and supersonic flow. Comparing equation 29 with equation 1, the transonic small disturbance equation, we notice that only the non-linear coefficient $M_\infty^2(\gamma + 1)u$ distinguishes the two equations from each other.

This coefficient becomes negligible when the Mach number becomes less than .8 or greater than 1.2. With this consideration in mind, it was assumed that equation 1 would adequately describe the flow through the nozzle and that the finite element formulation developed for the non-lifting airfoil would apply to the Laval nozzle.

A. BOUNDARY CONDITIONS

Two solutions are possible for a converging-diverging nozzle: 1) Symmetric flow, where the flow is subsonic through the domain, except for a small supersonic region near the wall in the throat, and 2) Asymmetric solution, where the flow accelerates to sonic velocity in the throat and then continues to accelerate to supersonic velocity in the diverging section. Different boundary conditions apply for the two solutions. For the symmetric case, both inlet and exit velocities must be specified. Inlet and exit velocities are equivalent in the subsonic solution. The supersonic solution requires that only the inlet velocities be specified. If the exit velocities are also applied, the problem is overspecified and the solution may not converge.

Velocities at the inlet and exit are not uniform in the y direction, therefore the disturbances cannot be set to zero as in the case of the non-lifting airfoil. Boundary velocities must be calculated by solving equation 29 analytically.

Equation 29 is a linear equation which can be mapped to Laplace's equation,

$$\nabla^2 \phi = 0,$$

by letting $y' = \sqrt{(1 - M_\infty^2)}\, y$. Laplace's equation is easily solved for the case of the hyperbolic nozzle by transforming from cartesian coordinates to elliptic coordinates. This transformation simplifies the solution because the stream lines must be hyperbolas to follow the nozzle boundary and therefore follow the hyperbolic coordinate ν = constant.

If the elliptic coordinates μ and ν are chosen such that the curves μ = constant are ellipses and the ν curves are hyperbolas, then the velocity potential which satisfies Laplace's equation for a hyperbolic nozzle is simply

$$\phi = A\mu$$

where A is a constant of integration. The stream function is

$$\psi = A\nu$$

The transformation $w = \mu + i\nu = \cosh^{-1}(2z/a)$ gives rise to the elliptic coordinates

$$y = 1/2\ a \cosh\mu \cos\nu, \quad x = 1/2\ a \sinh\mu \sin\nu$$

$$r = 1/2\ a\ [\cosh^2\mu - \sin^2\nu]$$

$$r_1 = \sqrt{(y + a/2)^2 + x^2}$$

$$r_2 = \sqrt{(y - a/2)^2 + x^2}$$

Solving for μ and ν produces

$$\mu = \cosh^{-1}\ [(r_1 + r_2)/a]$$

$$\nu = \cos^{-1}\ [(r_1 + r_2)/a]$$

The nozzle boundary is defined by ν_0 = constant, which along with the equation for the nozzle wall in cartesian

coordinates, $y' = f(x)$, implicitly defines the constant a. Substituting for μ in the velocity potential produces,

$$= A \cosh^{-1} [(r_1 + r_2)/a]$$

from which the velocities may be determined.

$$u = \phi_x = \frac{A}{\sqrt{[(r_1 + r_2)/a]^2 - 1}} \left\{ a\frac{\partial r_1}{\partial x} + a\frac{\partial r_2}{\partial y} \right\}$$

$$v = \phi_y = \frac{A}{\sqrt{(\frac{r_1 + r_2}{a})^2 - 1}} \left\{ a\frac{\partial r_1}{\partial y} + a\frac{\partial r_2}{\partial y} \right\}$$

$$\frac{\partial r_1}{\partial} = \frac{x}{\sqrt{(y + a/_2)^2 + x^2}}$$

$$\frac{\partial r_2}{\partial x} = \frac{x}{\sqrt{(y + a/_2)^2 + x^2}}$$

$$\frac{\partial r_1}{\partial y} = \frac{y + a/2}{\sqrt{(y - a/_2)^2 + x^2}}$$

$$\frac{\partial r_2}{\partial y} = \frac{y - a/2}{\sqrt{(y - a/_2)^2 + x^2}}$$

The constant of integration A may be determined by specifying the flow rate through the nozzle, but when the inlet velocities are normalized with respect to the freestream velocity (U_∞) A is factored out of the problem.

Velocities for compressible flow can be solved by mapping back to the physical coordinate system (x,y plane).

Other boundary conditions are universal to both problems. These are:

$u = 0$ on the line of symmetry

$v = (1 + u)df/dx$ at the nozzle wall

F(x) defines the nozzle boundary in terms of a ratio of the throat semi-height as a function of x. The throat semi-height is taken to be 1 for convenience.

Pressure ratio, sound speed, and Mach number are calculated as before by equations 5 through 8.

VIII. DISCUSSION OF RESULTS

A. TIME INTEGRATION TO A STEADY STATE SOLUTION

As stated before, the Houbolt method of integration is stable for all time steps. Results of the test cases bear this out with the larger time steps providing the most rapid convergence to a steady state solution. Time steps were tried from t = .1 to t = 100. Time steps larger than t = 100 were not attempted because as t becomes too large the influence that the damping and mass matrices have on the effective stiffness matrix becomes negligible compared to the stiffness matrix. That is:

$$SE_{ij} = S_{ij} + 2SC_{ij}/\Delta t^2 + 6SM_{ij}/\Delta t$$

as $\Delta t \to \infty$

$$SE_{ij} \simeq S_{ij}$$

The starting solutions were chosen somewhat arbitrarily. $\phi_{j,2\Delta t}$ was chosen to satisfy the first iteration of the steady solution

$$S_{ij}\phi_{j,2\Delta t} = 0$$

when the non-linear term (u) in the coefficient

$$1 - M_\infty^2 - M_\infty^2(\gamma + 1)\,u$$

was set to zero. $\phi_{j,\Delta t}$ and $\phi_{j,0}$ were chosen as multiples of $\phi_{j,2\Delta t}$ and respectively they were

$$\phi_{j,\Delta t} = .5\phi_{j,2\Delta t}$$

$$\phi_{j,0} = 0$$

This starting procedure proved to be superior to chosing the first three vectors closer to the converged solution. If $\phi_{j,2\Delta t}$, $\phi_{j,\Delta t}$, and $\phi_{j,0}$ were chosen to be the last three time steps of the previous case, the solution oscillated and converged much slower than with the starting solutions chosen as above.

The stiffness, mass, and damping matrices were recalculated after each time step, using the under-relaxation technique described above. This was necessary to utilize the special assembly procedures invoked by STRANL-II to prevent the inadmissable influence of downwind nodes from propagating upstream in the supersonic region.

For barely critical flow ($M_\infty = .861$) and subsonic flow, an under-relaxation factor $\theta = 1$ (successive approximation) resulted in convergence to a steady state solution after only three time steps. Eleven time steps were required for the supercritical solution to converge using the same relaxation factor. Reducing θ to .5 increased the rate of convergence and the solution achieved steady state after six time steps. Figure 4 compares the steady state solution for a 6% thick circular arc airfoil at $M_\infty = .909$, using the same integration method, with the results obtained in Ref. [5]. Chan's results converged in 10 iterations after using the results from the barely critical flow as an initial guess to the

supercritical solution. Figure 4 is a plot of local Mach numbers at boundary nodes on the airfoil.

B. CONVERGING-DIVERGING NOZZLE

The nozzle chosen for the test cases was the two-dimensional Oswatitsch nozzle with the boundary defined as

$$y = 1 + \sqrt{.2(x - 2.5)^2}$$

where the throat at x = 2.5 has a semi-height of 1. The inlet was taken to be x = 0 and the exit was at x = 5. $M_\infty = .44$, the inlet Mach number on the nozzle center-line was chosen to yield sonic conditions in the throat.

Two solutions were possible for this inlet condition,- the symmetric solution and the asymmetric solution; but neither solution was achieved by the finite element method. Although the solution converged for the subsonic case in three iterations, center-line Mach numbers deviated significantly from both one-dimensional theory and from Oswatitsch's approximation [Fig. 5]. When the local Mach number M exceeded the inlet Mach number by approximately .2 ($M \geq .64$) the solution was invalid. Differences at the center part of the nozzle are due to an essentially incorrect free stream Mach number. Patching the solution at $x \simeq 1.5$ would improve the solution.

A second test case was run for the supersonic section of the nozzle with the inlet boundary on the sonic line. The exit boundary was left free to float. Here the solution was unstable and no meaningful results were obtained.

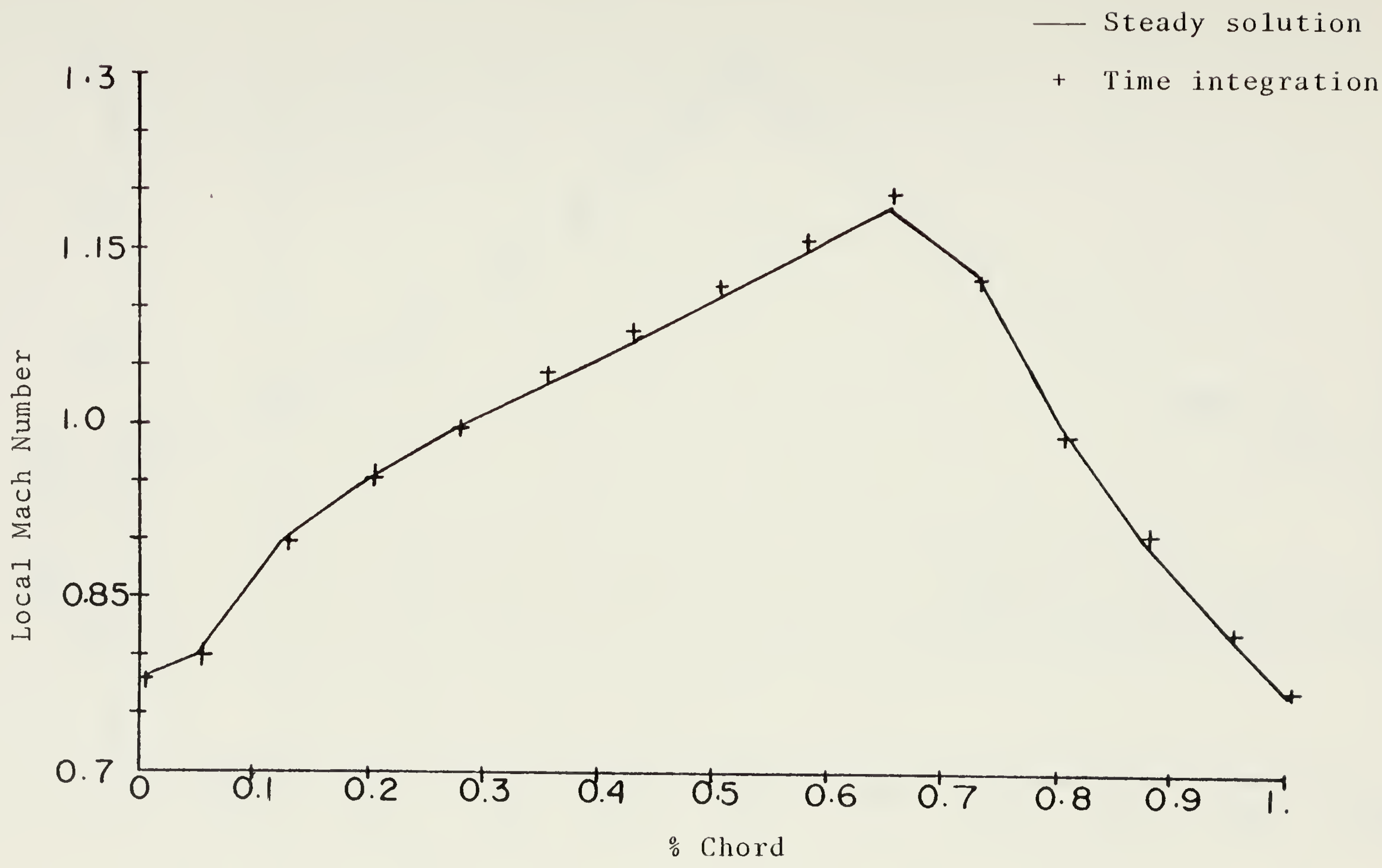


Figure 4 - Comparison of Time Integration Results with Steady State Results.

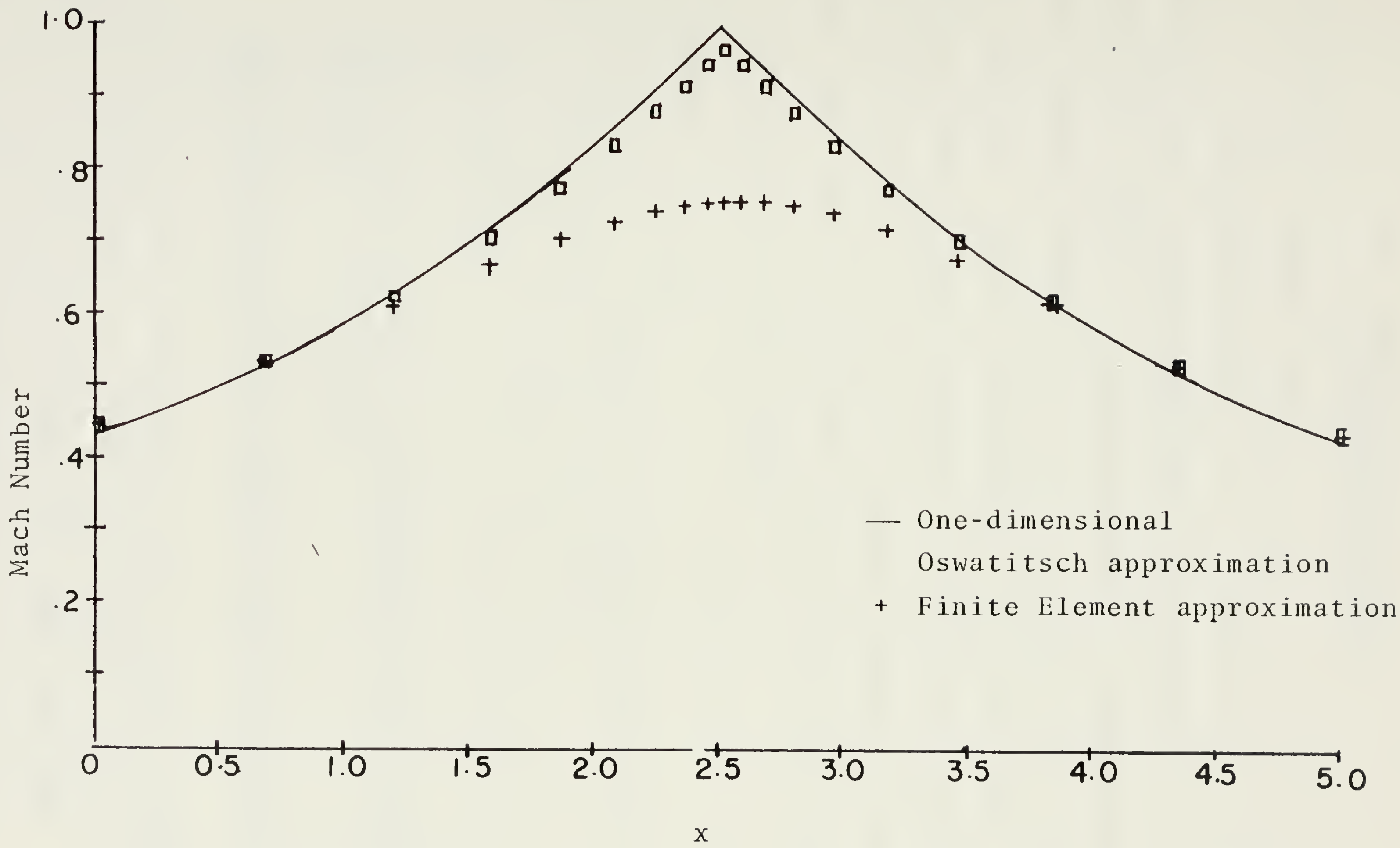


Figure 5 - Nozzle Center-Line, Inlet Mach Number 0.4300.

IX. PROGRAM MODIFICATION

The finite element computer program for non-lifting air-voils, as developed in Ref. [5], is separated into two parts. These have been designated STRANL-I and STRANL-II by Lockheed Corporation. STRANL-I generates a finite element mesh to be used as inputs to STRANL-II, which assembles the finite element equations, applies the boundary conditions, and solves the non-linear system of equations. Detailed descriptions and instructions for the use of the two programs can be found in Ref. [5]. Only the modifications to the above programs will be discussed in this section.

A. UNSTEADY EQUATIONS

Modifications to STRANL-II to form and solve the unsteady finite element equations were three-fold:

1) The new elemental matrices SC_{ij} and SM_{ij} were calculated and assembled.

2) All the matrices were stored on an external magnetic disk to be accessed and reassembled later because of the amount of space required to store three large matrices, in core memory.

3) The effective stiffness matrix SE_{ij} and the effective load vector RE_{ij} were assembled, and the system of equations solved.

Several existing subroutines in the original STRANL-II program were modified to assemble the damping and mass matrices. These include subroutines NEWK, EMTC, DERV, and EMQT. Two new subroutines were added to perform the other tasks.

B. MODIFICATIONS TO CALCULATE THE MASS AND DAMPING MATRICES

EMTC in the STRANL-II program calculated the elemental stiffness matrix by numerically integrating the equation,

$$S_{ij} = Q_i P_j \, dA$$

Equations were added to EMTC to perform the additional numerical integrations for the mass and damping matrices. All three matrices were calculated at the same time. EMQT assembled the elemental stiffness matrices for a quadrilateral and trapezoidal element from the contributions of the triangular elements. Mass and damping matrices were calculated in the same fashion.

Subroutine NEWK, an original subroutine in STRANL-II, which assembled the finite element equations for steady flow, was modified to assemble SC_{ij} and SM_{ij}. A new calling argument, NMAT was passed to NEWK, which assembled contributions from the triangular, quadrilateral and trapezoidal element matrices into the global matrices S_{ij}, SC_{ij}, and SM_{ij}, depending on NMAT being 1, 2 or 3.

1. Subroutine STORE

Given a non-symmetric matrix stored in a banded node, plus the right hand side vector, subroutine STORE separates this system into two matrices and stores them on a magnetic disk. Figures 6 and 7 show the decomposition of a banded matrix into banded storage, and the further decomposition of this banded stored matrix to two smaller matrices by subroutine STORE. In these figures, D, L, and U represent the

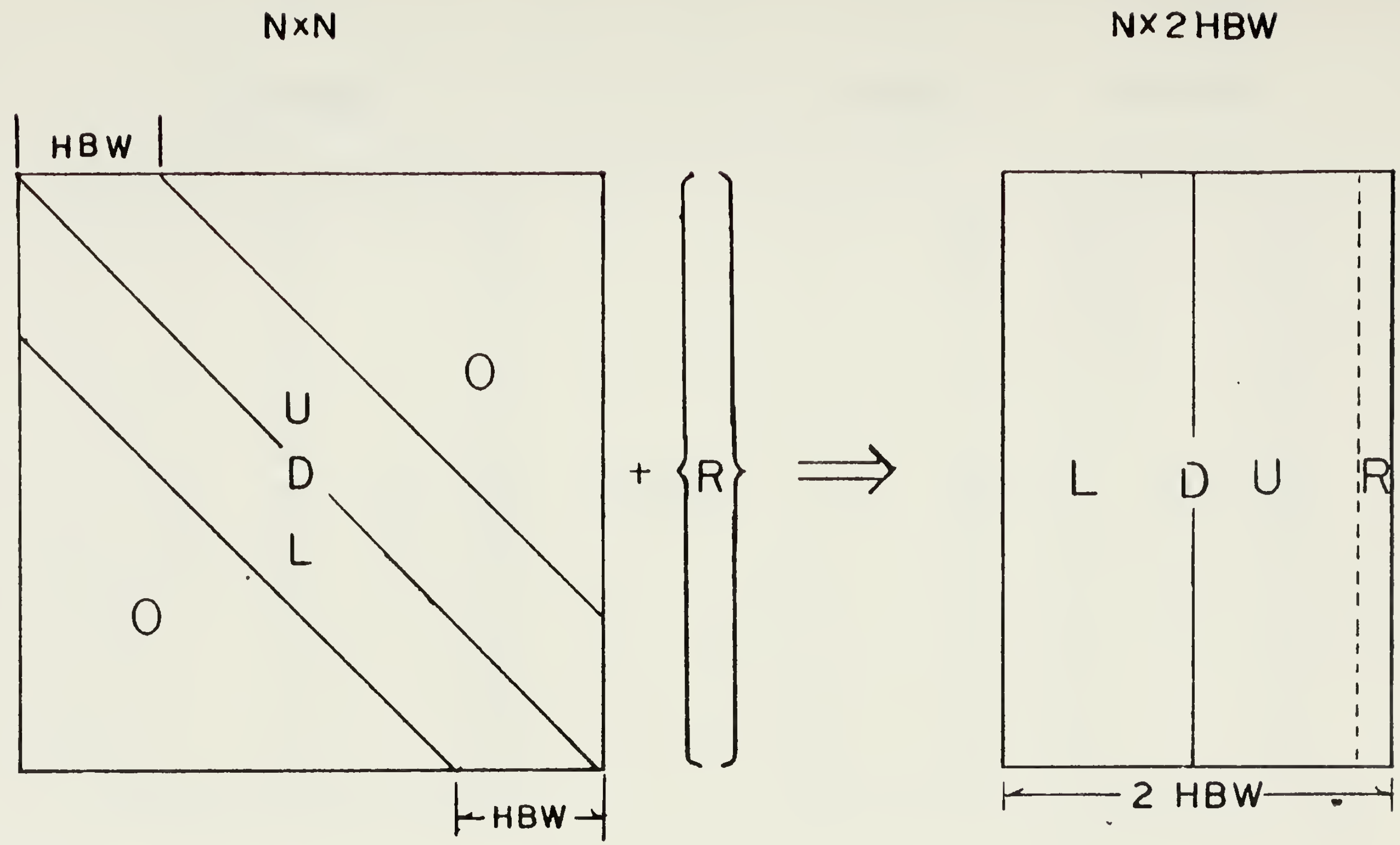


Figure 6 - Decomposition of a Banded Matrix Plus Right Hand Side

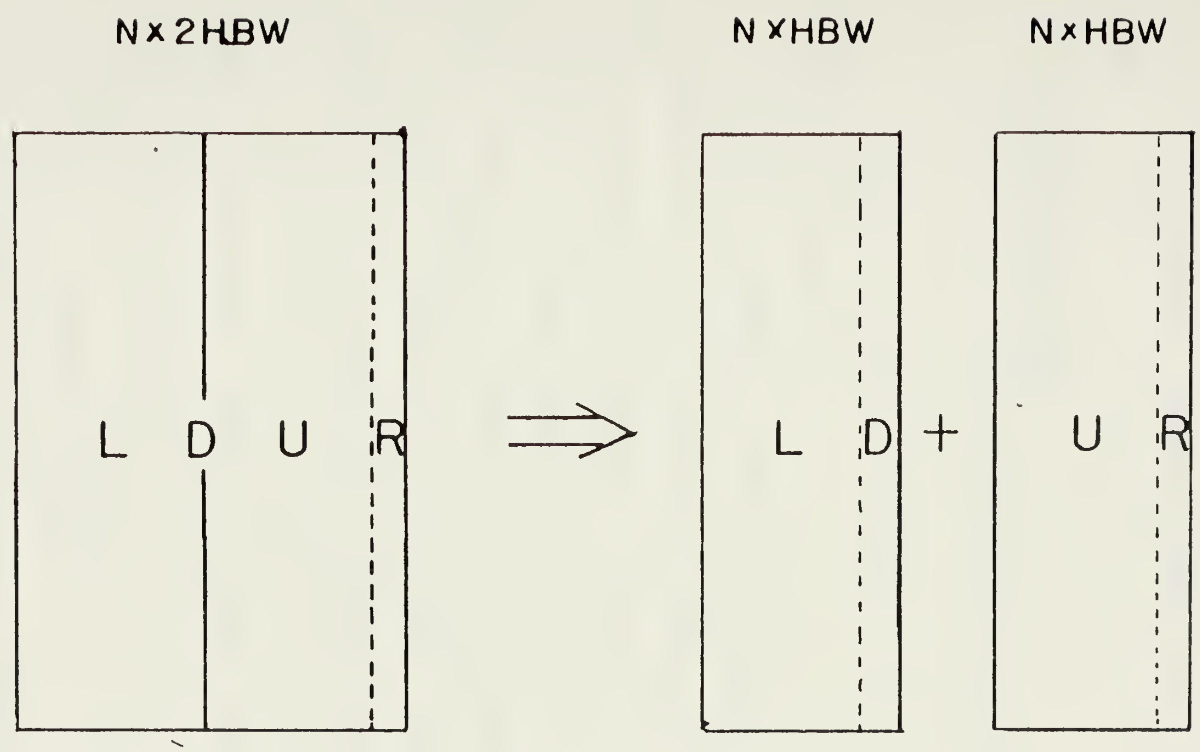


Figure 7 - Separation of a Banded Stored Matrix by Store

diagonal matrix, the lower triangular matrix, and the upper triangular matrix respectively. HBW is the half bandwidth and R is the right hand side vector.

STORE requires an additional work area one-half the size of the originally dimensioned matrix which is to be stored.

2. Subroutine TIME

Subroutine TIME integrates the system

$$S_{ij}\phi_j + SC_{ij}\dot{\phi}_j + SM_{ij}\ddot{\phi}_j = R_j$$

by Houbolt integration.

TIME reassembles the three matrices which were stored on the magnetic disk to form the effective stiffness matrix and the effective load vector. Once this system of equations is assembled, a banded equation solver is called to yield the solution for $\phi_{j,t+\Delta t}$. Figure 8 is a schematic flow chart of TIME. In Fig. 8 when L = 1, the lower triangular matrix and the diagonal of the effective stiffness matrix are formed by adding the appropriate contributions from the stiffness, mass and damping matrices. When L = 2, the upper triangular matrix is formed in like fashion.

C. CONVERGING-DIVERGING NOZZLE

1. Application of the Boundary Conditions

Regardless of the type of problem for which a set of system equations have been assembled, the equations will have the form

$$K_{ij}\ x_i = R_i$$

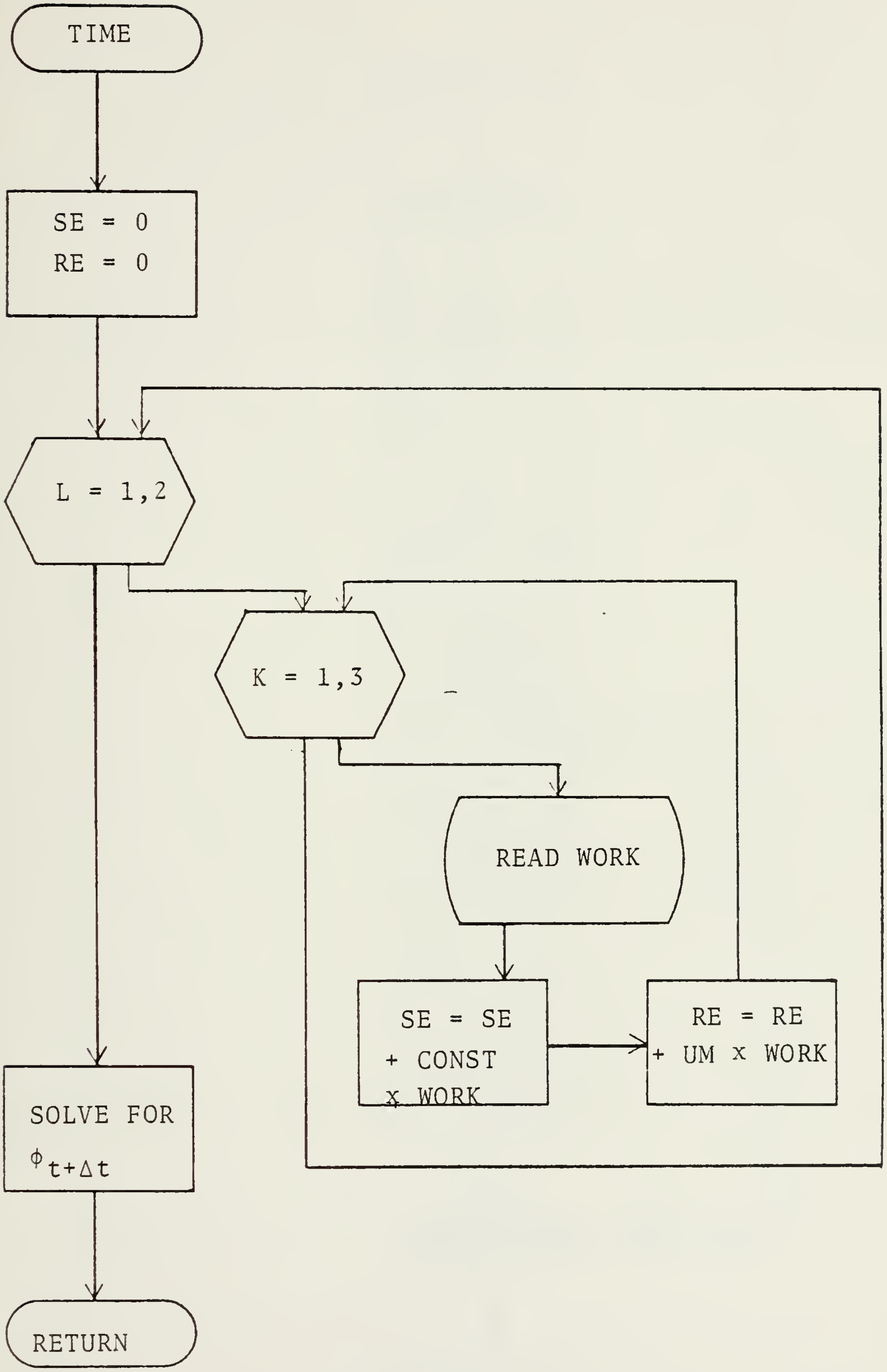


Figure 8 - Flow Chart of Subroutine TIME

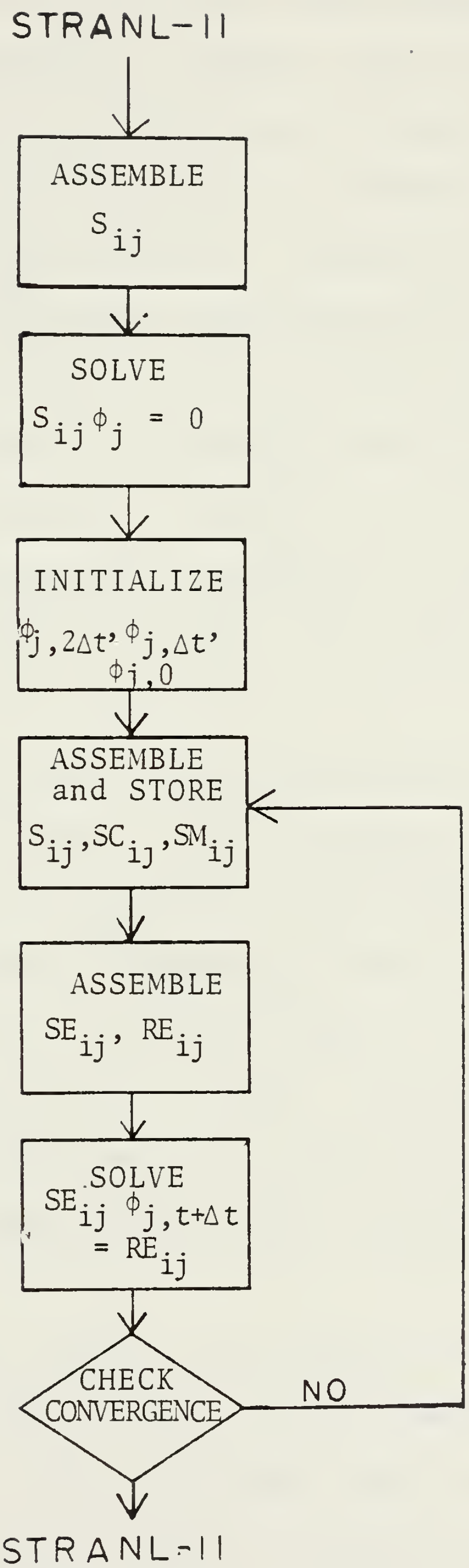


Figure 9 - Flow Chart of STANL-II Modification to Integrate Unsteady Equations

in which K_{ij} is an nxn matrix and x_i and R_i are vectors of length n. These equations do not take into account the known values of x_i on the boundaries. However, for a unique solution of the above equation, at least one or more nodal variables must be specified and K_{ij} must be modified to render it non-singular. For each equation i, either x_i or R_i must be specified but it is physically impossible to specify both x_i and R_i. There are a number of ways to apply the boundary conditions to the equations and when they are applied the number of equations is reduced. However, it is convenient to leave the number of equations unchanged to avoid major restructuring of the computer storage. One such method is described below.

If k is the subscript of the prescribed nodal variable, the k^{th} row and the k^{th} column of the original K_{ij} matrix are set to zero, K_{kk} is set to 1 and R_k is replaced by the known value of x_k. Each of the n-1 remaining terms of R_i is modified by subtracting from it the value $K_{ik}x_k$. This procedure is repeated for all the boundary values. Of course, when the matrix is stored in a banded mode, the algorithm will differ from that for the nxn square matrix, but the procedure is similar.

Subroutine BNDRY applies the boundary conditions for the modified program. Setting the option parameter IOPT(4), in STRANL-II, equal to 1 will call BNDRY which will read the boundary velocities and apply them to a banded stored matrix by the method described above.

When the value of the x_k is zero, as in the non-lifting airfoil problem, the algorithm becomes simpler than the above method because there is no need to either set the k^{th} column to zero or subtract the value $K_{ik} x_k$ from the right hand vector.

X. TEST CASES

In computing the flow field for either the non-lifting airfoil or for the Laval nozzle, the following procedures were followed:

1) The desired mesh was sketched with each node assigned a number.

2) Appropriate input cards based on the sketch were prepared and supplied to STRANL-I to generate the data on punched cards for input to STRANL-II.

3) The above punched cards were supplied to STRANL-II with three additional cards as input parameters for each case, plus additional cards for the boundary velocities, if the nozzle solution is desired.

4) Results of the finite element calculations are printed after each iteration, and the converged solution is punched on cards for possible later use.

A. TIME INTEGRATION TO A STEADY STATE SOLUTION

Test cases for the integration of the unsteady transonic finite element equations were conducted to calculate the steady transonic flow over a 6% thick circular arc airfoil. These tests were made using the same airfoil, mesh, and freestream Mach numbers as Chan et al. [Ref. 5] published. These conditions were chosen to provide a source for comparison of the results.

Freestream Mach numbers used in these calculations were:

M_∞ = .806 (subcritical)

M_∞ = .861 (barely critical)

M_∞ = .909 (supercritical).

Each case was treated individually with $\phi_j = 0$ used as the initial guess for each case, whereas Chan et al. [Ref. 5] used zero for the initial guess for $M_\infty = .806$ and then used the computed results as the initial guess for each subsequent case.

DELT, the value for the time step, is input by a parameter specified in columns 41-45 of the second card following the title card for each case when the unsteady option (IOPT(6) = 1) is selected.

1. <u>STRANL-I Program</u>

a. Input

Input cards used to generate the finite element mesh are listed on the next page. Cards were arranged in accordance with Ref. [5], in the following order:

Title card
Option card
Element cards
Card for the total number of nodes
Node coordinate cards
Card for the number of boundary nodes
Card for the boundary nodes at infinity
Card for the nodes on the line of symmetry
Card for the nodes on the airfoil
Cards for the slope of the airfoil.

Input cards to STRANL-I for these tests are listed on the next three pages.

b. Output

Output from STRANL-I is in the form of printouts and punched cards. Printouts from STRANL-I are listed on the following eight pages.

INPUT TO STRANL-I

```
STEADY TRANSONIC FLOW--MESH 6--154 ELEMENTS, 170 NODES                  TRA00010
                       1 1                                                TRA00020
   3   1   1   0   0  31  21  26                                          TRA00030
   4   5   4   5   1   6   1   2   7                                      TRA00040
   3   1   1   0   0  26  27  32                                          TRA00050
   4   3   1   1   0  27  28  33  32                                      TRA00060
   3   1   1   0   0  32  33  36                                          TRA00070
   4   2   5   1   3  33  34  37  36                                      TRA00080
   4   2   1   1   0  48  49  53  52                                      TRA00090
   3   1   1   0   0  48  52  51                                          TRA00100
   3   1   1   0   0  51  56  55                                          TRA00110
   4   3   1   1   0  51  52  57  56                                      TRA00120
   3   1   1   0   0  55  61  60                                          TRA00130
   4   4   1   1   0  55  56  62  61                                      TRA00140
   4   4   4   1   5  61  62  67  66                                      TRA00150
   4   1   1   0   0  10   5  86  87                                      TRA00160
   4   1   1   0   0  15  10  87  88                                      TRA00170
   4   1   1   0   0  20  15  88  92                                      TRA00180
   4   1   1   0   0  25  20  92  97                                      TRA00190
   4   1   1   0   0  30  25  97 103                                      TRA00200
   4   1   1   0   0  35  30 103 110                                      TRA00210
   4   1   1   0   0  38  35 110 117                                      TRA00220
   4   1   1   0   0  41  38 117 124                                      TRA00230
   4   1   1   0   0  44  41 124 131                                      TRA00240
   4   1   1   0   0  44 131 138  47                                      TRA00250
   4   1   1   0   0  47 138 145  50                                      TRA00260
   4   1   1   0   0  50 145 152  54                                      TRA00270
   4   1   1   0   0 152 158  59  54                                      TRA00280
   4   1   1   0   0 158 163  65  59                                      TRA00290
   4   1   1   0   0 163 167  70  65                                      TRA00300
   4   1   1   0   0 167 170  75  70                                      TRA00310
   4   1   1   0   0 170 169  80  75                                      TRA00320
   4   1   1   0   0 169 168  85  80                                      TRA00330
   4   2   1   1   0  87  86  89  90                                      TRA00340
   3   1   1   0   0  88  91  92                                          TRA00350
   4   3   1   1   0  90  89  93  94                                      TRA00360
   3   1   1   0   0  92  96  97                                          TRA00370
   4   4   1   1   0  94  93  98  99                                      TRA00380
   3   1   1   0   0  97 102 103                                          TRA00390
   4   5   1   1   0  99  98 104 105                                      TRA00400
   3   1   1   0   0 103 109 110                                          TRA00410
   4   6   6   1   7 105 104 111 112                                      TRA00420
   4   5   1   1   0 147 146 153 154                                      TRA00430
   3   1   1   0   0 152 151 158                                          TRA00440
   4   4   1   1   0 154 153 159 160                                      TRA00450
```

```
   3    1    1    0      0   158   157   163                                                        TRA00460
   4    3    1    1      0   160   159   164   165                                                  TRA00470
   3    1    1    0      0   163   162   167                                                        TRA00480
   4    2    1    1      0   165   164   168   169                                                  TRA00490
   3    1    1    0      0   167   166   170                                                        TRA00500
 (BLANK CARD)
 170                                                                                                TRA00520
   1    5    1   0.910        .00        0.00           1.80        0.00                             TRA00530
   6    5    1   0.910       0.00        0.30           1.81        0.10                             TRA00550
  11    5    1   0.910       0.00        0.70           1.82        0.26                             TRA00560
  16    5    1   0.870       0.00        1.10           1.86        0.43                             TRA00570
  21    5    1   0.834       0.00        1.50           1.92        0.66                             TRA00580
  31    1    0   1.00        0.00        2.00                                                        TRA00590
  26    5    1   0.910       0.50        2.00           2.02        0.86                             TRA00600
  32    4    1   0.910       1.20        2.00           2.12        1.08                             TRA00610
  36    3    1   0.870       1.80        2.00           2.25        1.15                             TRA00620
  39    3    1   0.870       2.20        2.00           2.38        1.22                             TRA00630
  42    3    1   0.910       2.50        2.00           2.50        1.25                             TRA00640
  45    3    1   0.870       2.80        2.00           2.62        1.22                             TRA00650
  48    3    1   0.870       3.20        2.00           2.75        1.15                             TRA00660
  51    4    1   0.910       3.80        2.00           2.88        1.06                             TRA00670
  55    5    1   0.910       4.50        2.00           2.98        0.86                             TRA00680
  60    1    0   1.00        5.00        2.00                                                        TRA00690
  61    5    1   0.834       5.00        1.50           3.08        0.66                             TRA00700
  66    5    1   0.870       5.00        1.10           3.12        0.46                             TRA00710
  71    5    1   0.910       5.00        0.70           3.15        0.26                             TRA00720
  76    5    1   0.910       5.00        0.30           3.20        0.10                             TRA00730
  81    5    1   0.910       5.00        0.00           3.20        0.00                             TRA00740
  86    3    1   1.20        2.00        0.00           2.00        0.22                             TRA00750
  89    4    1   1.20        2.05        0.00572        2.05        0.35                             TRA00760
  93    5    1   1.20        2.125       0.01315        2.125       0.55                             TRA00770
  98    6    1   1.20        2.20        0.01922        2.20        0.75                             TRA00780
 104    7    1   1.20        2.275       0.02394        2.275       0.92                             TRA00790
 111    7    1   1.20        2.35        0.02731        2.35        0.96                             TRA00800
 118    7    1   1.20        2.425       0.02933        2.425       1.00                             TRA00810
 125    7    1   1.20        2.50        0.03000        2.50        1.02                             TRA00820
 132    7    1   1.20        2.575       0.02933        2.575       1.00                             TRA00830
 139    7    1   1.20        2.65        0.02731        2.65        0.96                             TRA00840
 146    7    1   1.20        2.725       0.02394        2.725       0.92                             TRA00850
 153    6    1   1.20        2.80        0.01922        2.80        0.75
 159    5    1   1.20        2.875       0.01315        2.875       0.55                             TRA00870
 164    4    1   1.20        2.95        0.00572        2.95        0.35                             TRA00880
 168    3    1   1.20        3.00        0.00           3.00        0.22                             TRA00890
 (BLANK CARD)
  21    8   15                                                                                      TRA00910
   1    6   11   16     21    26    31    32    36    39    42    45    48    51                     55   60
  61   66   71   76     81
   2    3    4    5     82    83    84    85                                                        TRA00940
```

```
  86    89    93    98   104   111   118   125   132   139   146   153   159   164   168
   .06022  +0.10820  +0.09004  +0.07193  +0.05389  +0.03589  +0.01794  +0.00000
  -.01794  -0.03589  -0.05388  -0.07193  -0.09004  -0.10820  -0.06022
 1.0        8.0        0.1        3.5        0.1
CIRCLLAR ARC
```

OUTPUT FRCM STRANL-I

TIME INTEGRATION SOLUTION--MESH 6--154 ELEMENTS, 170 NODES

0 C C C 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0
ONO. OF ELEMENTS= 154 NO. OF NODES= 170 FULL BANCWIDTH= 84

ELE. NC. ANC ELEMENT NODES

1	31	21	26	0
2	6	1	2	7
3	11	6	7	12
4	16	11	12	17
5	21	16	17	22
6	26	21	22	27
7	7	2	3	8
8	12	7	8	13
9	17	12	13	18
10	22	17	18	23
11	27	22	23	28
12	8	3	4	9
13	13	8	9	14
14	18	13	14	19
15	23	18	19	24
16	28	23	24	29
17	9	4	5	10
18	14	9	10	15
19	19	14	15	20
20	24	19	20	25
21	29	24	25	30
22	26	27	32	0
23	27	28	33	32
24	28	29	34	33
25	29	30	35	34
26	32	33	36	0
27	33	34	37	36
28	34	35	38	37
29	36	37	40	39
30	37	38	41	40
31	39	40	43	42
32	40	41	44	43
33	42	43	46	45
34	43	44	47	46
35	45	46	49	48

36	46	47	50	49
37	48	49	53	52
38	49	50	54	53
39	48	52	51	0
40	51	56	55	0
41	51	52	57	56
42	52	53	58	57
43	53	54	59	58
44	55	61	60	0
45	55	56	62	61
46	56	57	63	62
47	57	58	64	63
48	58	59	65	64
49	61	62	67	66
50	62	63	68	67
51	63	64	69	68
52	64	65	70	69
53	66	67	72	71
54	67	68	73	72
55	68	69	74	73
56	69	70	75	74
57	71	72	77	76
58	72	73	78	77
59	73	74	79	78
60	74	75	80	79
61	76	77	82	81
62	77	78	83	82
63	78	79	84	83
64	79	80	85	84
65	10	5	86	87
66	15	10	87	88
67	20	15	88	92
68	25	20	92	97
69	30	25	97	103
70	35	30	103	110
71	38	35	110	117
72	41	38	117	124
73	44	41	124	131
74	44	131	138	47
75	47	138	145	50
76	50	145	152	54
77	152	158	59	54
78	158	163	65	59
79	163	167	70	65
80	167	170	75	70
81	170	169	80	75
82	169	168	85	80
83	87	86	89	90

84	88	87	90	91
85	88	91	92	0
86	90	89	93	94
87	91	90	94	95
88	92	91	95	96
89	92	96	97	0
90	94	93	98	99
91	95	94	99	100
92	96	95	100	101
93	97	96	101	102
94	97	102	103	0
95	99	98	104	105
96	100	99	105	106
97	101	100	106	107
98	102	101	107	108
99	103	102	108	109
100	103	109	110	0
101	105	104	111	112
102	106	105	112	113
103	107	106	113	114
104	108	107	114	115
105	109	108	115	116
106	110	109	116	117
107	112	111	118	119
108	113	112	119	120
109	114	113	120	121
110	115	114	121	122
111	116	115	122	123
112	117	116	123	124
113	119	118	125	126
114	120	119	126	127
115	121	120	127	128
116	122	121	128	129
117	123	122	129	130
118	124	123	130	131
119	126	125	132	133
120	127	126	133	134
121	128	127	134	135
122	129	128	135	136
123	130	129	136	137
124	131	130	137	138
125	133	132	139	140
126	134	133	140	141
127	135	134	141	142
128	136	135	142	143
129	137	136	143	144
130	138	137	144	145
131	140	139	146	147

132	141	140	147	148
133	142	141	148	149
134	143	142	149	150
135	144	143	150	151
136	145	144	151	152
137	147	146	153	154
138	148	147	154	155
139	149	148	155	156
140	150	149	156	157
141	151	150	157	158
142	152	151	158	0
143	154	153	159	160
144	155	154	160	161
145	156	155	161	162
146	157	156	162	163
147	158	157	163	0
148	160	159	164	165
149	161	160	165	166
150	162	161	166	167
151	163	162	167	0
152	165	164	168	169
153	166	165	169	170
154	167	166	170	0

OOLC NCCE	NEW NODE	X(I)	Y(I)
1	167	0.0	0.0
2	168	0.51551E 00	0.0
3	169	0.98463E 00	0.0
4	170	0.14115E 01	0.0
5	166	0.18000E 01	0.0
6	161	0.0	0.30000E 00
7	162	0.51838E 00	0.24272E 00
8	163	0.99010E 00	0.19060E 00
9	164	0.14194E 01	0.14316E 00
10	165	0.18100E 01	0.10000E 00
11	153	0.0	0.70000E 00
12	154	0.52124E 00	0.57399E 00
13	155	0.99557E 00	0.45931E 00
14	156	0.14272E 01	0.35496E 00
15	157	0.18200E 01	0.26000E 00
16	144	0.0	0.11000E 01
17	145	0.56614E 00	0.89607E 00
18	146	0.10587E 01	0.71865E 00
19	147	0.14872E 01	0.56429E 00
20	148	0.18600E 01	0.43000E 00
21	134	0.0	0.15000E 01
22	135	0.61743E 00	0.12299E 01
23	136	0.11324E 01	0.10046E 01

24	137	0.15618E 01	0.81670E 00
25	138	0.19200E 01	0.66000E 00
26	122	0.50000E 00	0.20000E 01
27	123	0.93532E 00	0.16735E 01
28	124	0.13315E 01	0.13764E 01
29	125	0.16920E 01	0.11060E 01
30	126	0.20200E 01	0.86000E 00
31	133	0.0	0.20000E 01
32	111	0.12000E 01	0.20000E 01
33	112	0.15360E 01	0.16640E 01
34	113	0.18418E 01	0.13582E 01
35	114	0.21200E 01	0.10800E 01
36	101	0.18000E 01	0.20000E 01
37	102	0.20406E 01	0.15455E 01
38	103	0.22500E 01	0.11500E 01
39	91	0.22000E 01	0.20000E 01
40	92	0.22963E 01	0.15829E 01
41	93	0.23800E 01	0.12200E 01
42	81	0.25000E 01	0.20000E 01
43	82	0.25000E 01	0.16073E 01
44	83	0.25000E 01	0.12500E 01
45	71	0.28000E 01	0.20000E 01
46	72	0.27037E 01	0.15829E 01
47	73	0.26200E 01	0.12200E 01
48	61	0.32000E 01	0.20000E 01
49	62	0.29594E 01	0.15455E 01
50	63	0.27500E 01	0.11500E 01
51	50	0.38000E 01	0.20000E 01
52	51	0.34640E 01	0.16567E 01
53	52	0.31582E 01	0.13443E 01
54	53	0.28800E 01	0.10600E 01
55	38	0.45000E 01	0.20000E 01
56	40	0.40647E 01	0.16735E 01
57	41	0.36685E 01	0.13764E 01
58	42	0.33080E 01	0.11060E 01
59	43	0.29800E 01	0.86000E 00
60	39	0.50000E 01	0.20000E 01
61	28	0.50000E 01	0.15000E 01
62	29	0.43826E 01	0.12299E 01
63	30	0.38676E 01	0.10046E 01
64	31	0.34382E 01	0.81670E 00
65	32	0.30800E 01	0.66000E 00
66	16	0.50000E 01	0.11000E 01
67	17	0.44278E 01	0.90520E 00
68	19	0.39299E 01	0.73572E 00
69	21	0.34968E 01	0.58828E 00
70	23	0.31200E 01	0.46000E 00
71	18	0.50000E 01	0.70000E 00

72	7	0.44702E 01	0.57399E 00
73	8	0.39880E 01	0.45931E 00
74	10	0.35493E 01	0.35496E 00
75	12	0.31500E 01	0.26000E 00
76	20	0.50000E 01	0.30000E 00
77	9	0.44845E 01	0.24272E 00
78	2	0.40154E 01	0.19060E 00
79	3	0.35885E 01	0.14316E 00
80	5	0.32000E 01	0.10000E 00
81	22	0.50000E 01	0.0
82	11	0.44845E 01	0.0
83	4	0.40154E 01	0.0
84	1	0.35885E 01	0.0
85	6	0.32000E 01	0.0
86	160	0.20000E 01	0.0
87	159	0.20000E 01	0.10000E 00
88	158	0.20000E 01	0.22000E 00
89	152	0.20500E 01	0.57200E-02
90	151	0.20500E 01	0.10030E 00
91	150	0.20500E 01	0.21380E 00
92	149	0.20500E 01	0.35000E 00
93	143	0.21250E 01	0.13150E-01
94	142	0.21250E 01	0.11316E 00
95	141	0.21250E 01	0.23317E 00
96	140	0.21250E 01	0.37718E 00
97	139	0.21250E 01	0.55000E 00
98	132	0.22000E 01	0.19220E-01
99	131	0.22000E 01	0.11742E 00
100	130	0.22000E 01	0.23526E 00
101	129	0.22000E 01	0.37668E 00
102	128	0.22000E 01	0.54637E 00
103	127	0.22000E 01	0.75000E 00
104	121	0.22750E 01	0.23940E-01
105	120	0.22750E 01	0.11418E 00
106	119	0.22750E 01	0.22246E 00
107	118	0.22750E 01	0.35241E 00
108	117	0.22750E 01	0.50834E 00
109	116	0.22750E 01	0.69546E 00
110	115	0.22750E 01	0.92000E 00
111	110	0.23500E 01	0.27310E-01
112	109	0.23500E 01	0.12124E 00
113	108	0.23500E 01	0.23395E 00
114	107	0.23500E 01	0.36921E 00
115	106	0.23500E 01	0.53151E 00
116	105	0.23500E 01	0.72628E 00
117	104	0.23500E 01	0.96000E 00
118	100	0.24250E 01	0.29330E-01
119	99	0.24250E 01	0.12708E 00

120	98	0.24250E 01	0.24438E 00
121	97	0.24250E 01	0.38515E 00
122	96	0.24250E 01	0.55406E 00
123	95	0.24250E 01	0.75676E 00
124	94	0.24250E 01	0.10000E 01
125	90	0.25000E 01	0.30000E-01
126	89	0.25000E 01	0.12970E 00
127	88	0.25000E 01	0.24934E 00
128	87	0.25000E 01	0.39290E 00
129	86	0.25000E 01	0.56518E 00
130	85	0.25000E 01	0.77192E 00
131	84	0.25000E 01	0.10200E 01
132	80	0.25750E 01	0.29330E-01
133	79	0.25750E 01	0.12708E 00
134	78	0.25750E 01	0.24438E 00
135	77	0.25750E 01	0.38515E 00
136	76	0.25750E 01	0.55406E 00
137	75	0.25750E 01	0.75676E 00
138	74	0.25750E 01	0.10000E 01
139	70	0.26500E 01	0.27310E-01
140	69	0.26500E 01	0.12124E 00
141	68	0.26500E 01	0.23395E 00
142	67	0.26500E 01	0.36921E 00
143	66	0.26500E 01	0.53151E 00
144	65	0.26500E 01	0.72628E 00
145	64	0.26500E 01	0.96000E 00
146	60	0.27250E 01	0.23940E-01
147	59	0.27250E 01	0.11418E 00
148	58	0.27250E 01	0.22246E 00
149	57	0.27250E 01	0.35241E 00
150	56	0.27250E 01	0.50834E 00
151	55	0.27250E 01	0.69546E 00
152	54	0.27250E 01	0.92000E 00
153	49	0.28000E 01	0.19220E-01
154	48	0.28000E 01	0.11742E 00
155	47	0.28000E 01	0.23526E 00
156	46	0.28000E 01	0.37668E 00
157	45	0.28000E 01	0.54637E 00
158	44	0.28000E 01	0.75000E 00
159	37	0.28750E 01	0.13150E-01
160	36	0.28750E 01	0.11316E 00
161	35	0.28750E 01	0.23317E 00
162	34	0.28750E 01	0.37718E 00
163	33	0.28750E 01	0.55000E 00
164	27	0.29500E 01	0.57200E-02
165	26	0.29500E 01	0.10030E 00
166	25	0.29500E 01	0.21380E 00
167	24	0.29500E 01	0.35000E 00

```
   168          15    0.30000E 01     0.0
   169          14    0.30000E 01     0.10000E 00
   170          13    0.30000E 01     0.22000E 00
 NODES AT FARFIELD
    1     6    11    16    21    26    31    32    36    39    42    45    48    51    55    60
   61    66    71    76    81
 NODES ON LINE OF SYMMETRY
    2     3     4     5    82    83    84    85
 NODES ON THE AIRFOIL
   86    89    93    98   104   111   118   125   132   139   146   153   159   164   168
 SLOPE ALONG NODES ON THE AIRFOIL
    0.60220E-01    0.10820E 00    0.90040E-01    0.71930E-01    0.53890E-01
    0.35890E-01    0.17940E-01    0.0
   -0.17940E-01   -0.35890E-01   -0.53880E-01   -0.71930E-01   -0.90040E-01
   -0.10820E 00   -0.60220E-01
```

2. STRANL-II Program

a. Input

Listed on the next three pages are the input cards to the STANL-II program.

b. Output

The output from this program is in the form of printouts for each iteration and punched cards for the converged solution. Output from STRANL-II for M_∞ = .909 is listed on the following eight pages.

INPUT TO STRANL-II

```
TIME INTEGRATION SOLUTICN -- CIRCULAR ARC -- 170 NODES -- M=0.909
0 0 0
   8   0.005        0.909       .500
 154   170    84    21     8    15
  31    21    26     0     6     1     2     7    11     6     7    12    16    11    12    17
  21    16    17    22    26    21    22    27     7     2     3     8    12     7     8    13
  17    12    13    18    22    17    18    23    27    22    23    28     8     3     4     9
  13     8     9    14    18    13    14    19    23    18    19    24    28    23    24    29
   9     4     5    10    14     9    10    15    19    14    15    20    24    19    20    25
  29    24    25    30    26    27    32     0    27    28    33    32    28    29    34    33
  29    30    35    34    32    33    36     0    33    34    37    36    34    35    38    37
  36    37    40    39    37    38    41    40    39    40    43    42    40    41    44    43
  42    43    46    45    43    44    47    46    45    46    49    48    46    47    50    49
  48    49    53    52    49    50    54    53    48    52    51     0    51    56    55     0
  51    52    57    56    52    53    58    57    53    54    59    58    55    61    60     0
  55    56    62    61    56    57    63    62    57    58    64    63    58    59    65    64
  61    62    67    66    62    63    68    67    63    64    69    68    64    65    70    69
  66    67    72    71    67    68    73    72    68    69    74    73    69    70    75    74
  71    72    77    76    72    73    78    77    73    74    79    78    74    75    80    79
  76    77    82    81    77    78    83    82    78    79    84    83    79    80    85    84
  10     5    86    87    15    10    87    88    20    15    88    92    25    20    92    97
  30    25    97   103    35    30   103   110    38    35   110   117    41    38   117   124
  44    41   124   131    44   131   138    47    47   138   145    50    50   145   152    54
 152   158    59    54   158   163    65    59   163   167    70    65   167   170    75    70
 170   169    80    75   169   168    85    80    87    86    89    90    88    87    90    91
  88    91    92     0    90    89    93    94    91    90    94    95    92    91    95    96
  92    96    97     0    94    93    98    99    95    94    99   100    96    95   100   101
  97    96   101   102    97   102   103     0    99    98   104   105   100    99   105   106
 101   100   106   107   102   101   107   108   103   102   108   109   103   109   110     0
 105   104   111   112   106   105   112   113   107   106   113   114   108   107   114   115
 109   108   115   116   110   109   116   117   112   111   118   119   113   112   119   120
 114   113   120   121   115   114   121   122   116   115   122   123   117   116   123   124
 119   118   125   126   120   119   126   127   121   120   127   128   122   121   128   129
 123   122   129   130   124   123   130   131   126   125   132   133   127   126   133   134
 128   127   134   135   129   128   135   136   130   129   136   137   131   130   137   138
 133   132   139   140   134   133   140   141   135   134   141   142   136   135   142   143
 137   136   143   144   138   137   144   145   140   139   146   147   141   140   147   148
 142   141   148   149   143   142   149   150   144   143   150   151   145   144   151   152
 147   146   153   154   148   147   154   155   149   148   155   156   150   149   156   157
 151   150   157   158   152   151   158     0   154   153   159   160   155   154   160   161
 156   155   161   162   157   156   162   163   158   157   163     0   160   159   164   165
 161   160   165   166   162   161   166   167   163   162   167     0   165   164   168   169
 166   165   169   170   167   166   170     0
  0.0         0.0         5.155E-01 0.0         9.846E-01 0.0         1.412E 00 0.0
  1.800E 00   0.0         0.0         3.000E-01 5.184E-01 2.427E-01 9.901E-01 1.906E-01
```

```
 1.419E 00 1.432E-01 1.810E 00 1.000E-01 0.0       7.000E-01 5.212E-01 5.740E-01
 9.956E-01 4.593E-01 1.427E 00 3.550E-01 1.820E 00 2.600E-01 0.0       1.100E 00
 5.661E-01 8.961E-01 1.059E 00 7.186E-01 1.487E 00 5.643E-01 1.860E 00 4.300E-01
 0.0       1.500E 00 6.174E-01 1.230E 00 1.132E 00 1.005E 00 1.562E 00 8.167E-01
 1.920E 00 6.600E-01 5.000E-01 2.000E 00 9.353E-01 1.674E 00 1.331E 00 1.376E 00
 1.692E 00 1.106E 00 2.020E 00 8.600E-01 0.0       2.000E 00 1.200E 00 2.000E 00
 1.536E 00 1.664E 00 1.842E 00 1.358E 00 2.120E 00 1.080E 00 1.800E 00 2.000E 00
 2.041E 00 1.545E 00 2.250E 00 1.150E 00 2.200E 00 2.000E 00 2.296E 00 1.583E 00
 2.380E 00 1.220E 00 2.500E 00 2.000E 00 2.500E 00 1.607E 00 2.500E 00 1.250E 00
 2.800E 00 2.000E 00 2.704E 00 1.583E 00 2.620E 00 1.220E 00 3.200E 00 2.000E 00
 2.959E 00 1.545E 00 2.750E 00 1.150E 00 3.800E 00 2.000E 00 3.464E 00 1.657E 00
 3.158E 00 1.344E 00 2.880E 00 1.060E 00 4.500E 00 2.000E 00 4.065E 00 1.674E 00
 3.669E 00 1.376E 00 3.308E 00 1.106E 00 2.980E 00 8.600E-01 5.000E 00 2.000E 00
 5.000E 00 1.500E 00 4.383E 00 1.230E 00 3.868E 00 1.005E 00 3.438E 00 8.167E-01
 3.080E 00 6.600E-01 5.000E 00 1.100E 00 4.428E 00 9.052E-01 3.930E 00 7.357E-01
 3.497E 00 5.883E-01 3.120E 00 4.600E-01 5.000E 00 7.000E-01 4.470E 00 5.740E-01
 3.988E 00 4.593E-01 3.549E 00 3.550E-01 3.150E 00 2.600E-01 5.000E 00 3.000E-01
 4.484E 00 2.427E-01 4.015E 00 1.906E-01 3.588E 00 1.432E-01 3.200E 00 1.000E-01
 5.000E 00 0.0       4.484E 00 0.0       4.015E 00 0.0       3.588E 00 0.0
 3.200E 00 0.0       2.000E 00 0.0       2.000E 00 1.000E-01 2.000E 00 2.200E-01
 2.050E 00 5.720E-03 2.050E 00 1.003E-01 2.050E 00 2.138E-01 2.050E 00 3.500E-01
 2.125E 00 1.315E-02 2.125E 00 1.132E-01 2.125E 00 2.332E-01 2.125E 00 3.772E-01
 2.125E 00 5.500E-01 2.200E 00 1.922E-02 2.200E 00 1.174E-01 2.200E 00 2.353E-01
 2.200E 00 3.767E-01 2.200E 00 5.464E-01 2.200E 00 7.500E-01 2.275E 00 2.394E-02
 2.275E 00 1.142E-01 2.275E 00 2.225E-01 2.275E 00 3.524E-01 2.275E 00 5.083E-01
 2.275E 00 6.955E-01 2.275E 00 9.200E-01 2.350E 00 2.731E-02 2.350E 00 1.212E-01
 2.350E 00 2.340E-01 2.350E 00 3.692E-01 2.350E 00 5.315E-01 2.350E 00 7.263E-01
 2.350E 00 9.600E-01 2.425E 00 2.933E-02 2.425E 00 1.271E-01 2.425E 00 2.444E-01
 2.425E 00 3.851E-01 2.425E 00 5.541E-01 2.425E 00 7.568E-01 2.425E 00 1.000E 00
 2.500E 00 3.000E-02 2.500E 00 1.297E-01 2.500E 00 2.493E-01 2.500E 00 3.929E-01
 2.500E 00 5.652E-01 2.500E 00 7.719E-01 2.500E 00 1.020E 00 2.575E 00 2.933E-02
 2.575E 00 1.271E-01 2.575E 00 2.444E-01 2.575E 00 3.851E-01 2.575E 00 5.541E-01
 2.575E 00 7.568E-01 2.575E 00 1.000E 00 2.650E 00 2.731E-02 2.650E 00 1.212E-01
 2.650E 00 2.340E-01 2.650E 00 3.692E-01 2.650E 00 5.315E-01 2.650E 00 7.263E-01
 2.650E 00 9.600E-01 2.725E 00 2.394E-02 2.725E 00 1.142E-01 2.725E 00 2.225E-01
 2.725E 00 3.524E-01 2.725E 00 5.083E-01 2.725E 00 6.955E-01 2.725E 00 9.200E-01
 2.800E 00 1.922E-02 2.800E 00 1.174E-01 2.800E 00 2.353E-01 2.800E 00 3.767E-01
 2.800E 00 5.464E-01 2.800E 00 7.500E-01 2.875E 00 1.315E-02 2.875E 00 1.132E-01
 2.875E 00 2.332E-01 2.875E 00 3.772E-01 2.875E 00 5.500E-01 2.950E 00 5.720E-03
 2.950E 00 1.003E-01 2.950E 00 2.138E-01 2.950E 00 3.500E-01 3.000E 00 0.0
 3.000E 00 1.000E-01 3.000E 00 2.200E-01
     1    6   11   16   21   26   31   32   36   39   42   45   48   51   55   60
    61   66   71   76   81
     2    3    4    5   82   83   84   85
    86   89   93   98  104  111  118  125  132  139  146  153  159  164  168
 6.022E-02 1.082E-01 9.004E-02 7.193E-02 5.389E-02 3.589E-02 1.794E-02 0.0
-1.794E-02-3.589E-02-5.388E-02-7.193E-02-9.004E-02-1.082E-01-6.022E-02
   167  168  169  170  166  161  162  163  164  165  153  154  155  156  157  144
```

145	146	147	148	134	135	136	137	138	122	123	124	125	126	133	111
112	113	114	101	102	103	91	92	93	81	82	83	71	72	73	61
62	63	50	51	52	53	38	40	41	42	43	39	28	29	30	31
32	16	17	19	21	23	18	7	8	10	12	20	9	2	3	5
22	11	4	1	6	160	159	158	152	151	150	149	143	142	141	140
139	132	131	130	129	128	127	121	120	119	118	117	116	115	110	109
108	107	106	105	104	100	99	98	97	96	95	94	90	89	88	87
86	85	84	80	79	78	77	76	75	74	70	69	68	67	66	65
64	60	59	58	57	56	55	54	49	48	47	46	45	44	37	36
35	34	33	27	26	25	24	15	14	13						

TIME INTEGRATION TO STEADY SOLUTION — CIRCULAR ARC --170 NODES--M=0.909
CONVERGENCE LIMIT =0.0050

0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0

NO. OF ELEMENTS= 154 NO. OF NODES= 170 FULL BANDWITH = 84

ELE. NO. AND ELEMENT NODES

1	31	21	26	0
2	6	1	2	7
3	11	6	7	12
4	16	11	12	17
5	21	16	17	22
6	26	21	22	27
7	7	2	3	8
8	12	7	8	13
9	17	12	13	18
10	22	17	18	23
11	27	22	23	28
12	8	3	4	9
13	13	8	9	14
14	18	13	14	19
15	23	18	19	24
16	28	23	24	29
17	9	4	5	10
18	14	9	10	15
19	19	14	15	20
20	24	19	20	25
21	29	24	25	30
22	26	27	32	0
23	27	28	33	32
24	28	29	34	33
25	29	30	35	34
26	32	33	36	0
27	33	34	37	36
28	34	35	38	37
29	36	37	40	39
30	37	38	41	40
31	39	40	43	42
32	40	41	44	43
33	42	43	46	45
34	43	44	47	46
35	45	46	49	48
36	46	47	50	49
37	48	49	53	52
38	49	50	54	53
39	48	52	51	0
40	51	56	55	0
41	51	52	57	56
42	52	53	58	57
43	53	54	59	58
44	55	61	60	0
45	55	56	62	61
46	56	57	63	62
47	57	58	64	63
48	58	59	65	64
49	61	62	67	66
50	62	63	68	67
51	63	64	69	68
52	64	65	70	69
53	66	67	72	71
54	67	68	73	72
55	68	69	74	73
56	69	70	75	74
57	71	72	77	76
58	72	73	78	77
59	73	74	79	78
60	74	75	80	79
61	76	77	82	81
62	77	78	83	82
63	78	79	84	83
64	79	80	85	84
65	10	5	86	87
66	15	10	87	88
67	20	15	88	92
68	25	20	92	97
69	30	25	97	103
70	35	30	103	110
71	38	35	110	117
72	41	38	117	124
73	44	41	124	131
74	44	131	138	47
75	47	138	145	50
76	50	145	152	54
77	152	158	59	54
78	158	163	65	59
79	163	167	70	65
80	167	170	75	70
81	170	169	80	75
82	169	168	85	80
83	87	86	89	90
84	88	87	90	91
85	88	91	92	0
86	90	89	93	94
87	91	90	94	95
88	92	91	95	96

89	92	96	97	0
90	94	93	98	99
91	95	94	99	100
92	96	95	100	101
93	97	96	101	102
94	97	102	103	0
95	99	98	104	105
96	100	99	105	106
97	101	100	106	107
98	102	101	107	108
99	103	102	108	109
100	103	109	110	0
101	105	104	111	112
102	106	105	112	113
103	107	106	113	114
104	108	107	114	115
105	109	108	115	116
106	110	109	116	117
107	112	111	118	119
108	113	112	119	120
109	114	113	120	121
110	115	114	121	122
111	116	115	122	123
112	117	116	123	124
113	119	118	125	126
114	120	119	126	127
115	121	120	127	128
116	122	121	128	129
117	123	122	129	130
118	124	123	130	131
119	126	125	132	133
120	127	126	133	134
121	128	127	134	135
122	129	128	135	136
123	130	129	136	137
124	131	130	137	138
125	133	132	139	140
126	134	133	140	141
127	135	134	141	142
128	136	135	142	143
129	137	136	143	144
130	138	137	144	145
131	140	139	146	147
132	141	140	147	148
133	142	141	148	149
134	143	142	149	150
135	144	143	150	151
136	145	144	151	152
137	147	146	153	154
138	148	147	154	155
139	149	148	155	156
140	150	149	156	157
141	151	150	157	158
142	152	151	158	0
143	154	153	159	160
144	155	154	160	161
145	156	155	161	162
146	157	156	162	163
147	158	157	163	0
148	160	159	164	165
149	161	160	165	166
150	162	161	166	167
151	163	162	167	0
152	165	164	168	169
153	166	165	169	170
154	167	166	170	0

OLD NODE	NEW NODE	X(I)Y(I)	
1	167	0.0	0.0
2	168	0.51550E 00	0.0
3	169	0.98460E 00	0.0
4	170	0.14120E 01	0.0
5	166	0.18000E 01	0.0
6	161	0.0	0.30000E 00
7	162	0.51840E 00	0.24270E 00
8	163	0.99010E 00	0.19060E 00
9	164	0.14190E 01	0.14320E 00
10	165	0.18100E 01	0.10000E 00
11	153	0.0	0.70000E 00
12	154	0.52120E 00	0.57400E 00
13	155	0.99560E 00	0.45930E 00
14	156	0.14270E 01	0.35500E 00
15	157	0.18200E 01	0.26000E 00
16	144	0.0	0.11000E 01
17	145	0.56610E 00	0.89610E 00
18	146	0.10590E 01	0.71860E 00
19	147	0.14870E 01	0.56430E 00
20	148	0.18600E 01	0.43000E 00
21	134	0.0	0.15000E 01
22	135	0.61740E 00	0.12300E 01
23	136	0.11320E 01	0.10050E 01
24	137	0.15620E 01	0.81670E 00
25	138	0.19200E 01	0.66000E 00
26	122	0.53030E 00	0.20000E 01
27	123	0.93530E 00	0.16740E 01
28	124	0.13310E 01	0.13760E 01
29	125	0.16520E 01	0.11060E 01
30	126	0.20200E 01	0.36000E 00
31	133	0.0	0.20000E 01
32	111	0.12000E 01	0.20000E 01
33	112	0.15360E 01	0.16640E 01
34	113	0.18420E 01	0.13580E 01

35	114	0.21200E 01	0.10800E 01
36	101	0.18000E 01	0.20000E 01
37	102	0.20410E 01	0.15450E 01
38	103	0.22500E 01	0.11500E 01
39	91	0.22000E 01	0.20000E 01
40	92	0.22960E 01	0.15830E 01
41	93	0.23800E 01	0.12200E 01
42	81	0.25000E 01	0.20000E 01
43	82	0.25000E 01	0.16070E 01
44	83	0.25000E 01	0.12500E 01
45	71	0.28000E 01	0.20000E 01
46	72	0.27040E 01	0.15830E 01
47	73	0.26200E 01	0.12200E 01
48	61	0.32000E 01	0.20000E 01
49	62	0.29590E 01	0.15450E 01
50	63	0.27500E 01	0.11500E 01
51	50	0.38000E 01	0.20000E 01
52	51	0.34640E 01	0.16570E 01
53	52	0.31580E 01	0.13440E 01
54	53	0.28800E 01	0.10600E 01
55	38	0.45000E 01	0.20000E 01
56	40	0.40650E 01	0.16740E 01
57	41	0.36690E 01	0.13760E 01
58	42	0.33080E 01	0.11060E 01
59	43	0.29800E 01	0.86000E 00
60	39	0.50000E 01	0.20000E 01
61	28	0.50000E 01	0.15000E 01
62	29	0.43830E 01	0.12300E 01
63	30	0.38680E 01	0.10050E 01
64	31	0.34380E 01	0.81670E 00
65	32	0.30800E 01	0.66000E 00
66	16	0.50000E 01	0.11000E 01
67	17	0.44280E 01	0.90520E 00
68	19	0.39300E 01	0.73570E 00
69	21	0.34970E 01	0.58830E 00
70	23	0.31200E 01	0.46000E 00
71	18	0.50000E 01	0.70000E 00
72	7	0.44700E 01	0.57400E 00
73	8	0.39880E 01	0.45930E 00
74	10	0.35490E 01	0.35500E 00
75	12	0.31500E 01	0.26000E 00
76	20	0.50000E 01	0.30000E 00
77	9	0.44840E 01	0.24270E 00
78	2	0.40150E 01	0.19060E 00
79	3	0.35880E 01	0.14320E 00
80	5	0.32000E 01	0.10000E 00
81	22	0.50000E 01	0.0
82	11	0.44840E 01	0.0
83	4	0.40150E 01	0.0
84	1	0.35880E 01	0.0
85	6	0.32000E 01	0.0
86	160	0.20000E 01	0.0
87	159	0.20000E 01	0.10000E 00
88	158	0.20000E 01	0.22000E 00
89	152	0.20500E 01	0.57200E-02
90	151	0.20500E 01	0.10030E 00
91	150	0.20500E 01	0.21380E 00
92	149	0.20500E 01	0.35000E 00
93	143	0.21250E 01	0.13150E-01
94	142	0.21250E 01	0.11320E 00
95	141	0.21250E 01	0.23320E 00
96	140	0.21250E 01	0.37720E 00
97	139	0.21250E 01	0.55000E 00
98	132	0.22000E 01	0.19220E-01
99	131	0.22000E 01	0.11740E 00
100	130	0.22000E 01	0.23530E 00
101	129	0.22000E 01	0.37670E 00
102	128	0.22000E 01	0.54640E 00
103	127	0.22000E 01	0.75000E 00
104	121	0.22750E 01	0.23940E-01
105	120	0.22750E 01	0.11420E 00
106	119	0.22750E 01	0.22250E 00
107	118	0.22750E 01	0.35240E 00
108	117	0.22750E 01	0.50830E 00
109	116	0.22750E 01	0.69550E 00
110	115	0.22750E 01	0.92000E 00
111	110	0.23500E 01	0.27310E-01
112	109	0.23500E 01	0.12120E 00
113	108	0.23500E 01	0.23400E 00
114	107	0.23500E 01	0.36920E 00
115	106	0.23500E 01	0.53150E 00
116	105	0.23500E 01	0.72630E 00
117	104	0.23500E 01	0.96000E 00
118	100	0.24250E 01	0.29330E-01
119	99	0.24250E 01	0.12710E 00
120	98	0.24250E 01	0.24440E 00
121	97	0.24250E 01	0.38510E 00
122	96	0.24250E 01	0.55410E 00
123	95	0.24250E 01	0.75680E 00
124	94	0.24250E 01	0.10000E 01
125	90	0.25000E 01	0.30000E-01
126	89	0.25000E 01	0.12970E 00
127	88	0.25000E 01	0.24930E 00
128	87	0.25000E 01	0.39290E 00
129	86	0.25000E 01	0.56520E 00
130	85	0.25000E 01	0.77190E 00
131	84	0.25000E 01	0.10200E 01
132	80	0.25750E 01	0.29330E-01
133	79	0.25750E 01	0.12710E 00
134	78	0.25750E 01	0.24440E 00
135	77	0.25750E 01	0.38510E 00
136	76	0.25750E 01	0.55410E 00
137	75	0.25750E 01	0.75680E 00
138	74	0.25750E 01	0.10000E 01

139	70	0.26500E 01	0.27310E-01
140	69	0.26500E 01	0.12120E 00
141	68	0.26500E 01	0.23400E 00
142	67	0.26500E 01	0.36920E 00
143	66	0.26500E 01	0.53150E 00
144	65	0.26500E 01	0.72630E 00
145	64	0.26500E 01	0.96000E 00
146	60	0.27250E 01	0.23940E-01
147	59	0.27250E 01	0.11420E 00
148	58	0.27250E 01	0.22250E 00
149	57	0.27250E 01	0.35240E 00
150	56	0.27250E 01	0.50830E 00
151	55	0.27250E 01	0.69550E 00
152	54	0.27250E 01	0.92000E 00
153	49	0.28000E 01	0.19220E-01
154	48	0.28000E 01	0.11740E 00
155	47	0.28000E 01	0.23530E 00
156	46	0.28000E 01	0.37670E 00
157	45	0.28000E 01	0.54640E 00
158	44	0.28000E 01	0.75000E 00
159	37	0.28750E 01	0.13150E-01
160	36	0.28750E 01	0.11320E 00
161	35	0.28750E 01	0.23320E 00
162	34	0.28750E 01	0.37720E 00
163	33	0.28750E 01	0.55000E 00
164	27	0.29500E 01	0.57200E-02
165	26	0.29500E 01	0.10030E 00
166	25	0.29500E 01	0.21380E 00
167	24	0.29500E 01	0.35000E 00
168	15	0.30000E 01	0.0
169	14	0.30000E 01	0.10000E 00
170	13	0.30000E 01	0.22000E 00

```
NODES AT FARFIELD
   1     6    11    16    21    26    31    32    36    39    42    45    48    51    55    60    61    66    71    76
  81

NODES ON THE LINE OF SYMMETRY
   2     3     4     5    82    83    84    85

NODES ON THE AIRFOIL
  86    89    93    98   104   111   118   125   132   139   146   153   159   164   168

SLOPE ALONG NODES ON AIRFOIL
   0.60220E-01   0.10820E 00   0.90040E-01   0.71930E-01   0.53890E-01   0.35890E-01   0.17940E-01   0.0
  -0.17940E-01  -0.35850E-01  -0.53880E-01  -0.71930E-01  -0.90040E-01  -0.10820E 00  -0.60220E-01
```

MACH NUMBER= 0.909 RELAX. FACTOR =0.5000

NO. OF ITERATIONS = 7

NODE	PHI	UCOM	VCOM	CEF	LMAC	P/PC	CP	DELM
1	1.7024E-05	1.0000E 00	-3.9050E-06	8.2629E-01	9.0900E-01	5.8550E-01	-7.2697E-06	0.0
2	4.4124E-04	1.0018E 00	-1.4605E-06	8.2986E-01	9.1092E-01	5.8427E-01	-3.6327E-03	-1.1861E-05
3	-6.6919E-04	9.9403E-01	8.4095E-05	8.1444E-01	9.0269E-01	5.8954E-01	1.1541E-02	-1.8382E-04
4	-5.3321E-03	9.8423E-01	2.9021E-04	7.9501E-01	8.9236E-01	5.9616E-01	3.1542E-02	-7.7355E-04
5	-2.3599E-02	9.2431E-01	5.8507E-04	6.7618E-01	8.3026E-01	6.3633E-01	1.5138E-01	-1.5438E-03
6	-1.3806E-05	1.0000E 00	6.7267E-07	8.2627E-01	9.0900E-01	5.8550E-01	8.7757E-06	0.0
7	3.8957E-04	1.0011E 00	-3.5716E-04	8.2847E-01	9.1017E-01	5.8475E-01	-2.2117E-03	-1.3053E-05
8	-6.6336E-04	9.9470E-01	2.2833E-04	8.1577E-01	9.0339E-01	5.8909E-01	1.0603E-02	-2.0063E-04
9	-5.4462E-03	9.8150E-01	-3.3073E-05	7.8960E-01	8.8949E-01	5.9801E-01	3.6596E-02	-7.6258E-04
10	-2.3760E-02	9.2877E-01	1.0484E-02	6.8503E-01	8.3469E-01	6.3332E-01	1.4246E-01	-1.5749E-03
11	1.4776E-06	1.0000E 00	1.3370E-06	8.2628E-01	9.0900E-01	5.8550E-01	1.5906E-06	0.0
12	2.7864E-04	1.0009E 00	-3.2061E-04	8.2812E-01	9.0998E-01	5.8437E-01	-1.8504E-03	0.0
13	-9.3074E-04	9.9653E-01	8.8734E-04	8.1949E-01	9.0533E-01	5.8785E-01	6.9402E-03	-1.7560E-04
14	-5.2681E-03	9.8220E-01	2.0128E-03	7.9098E-01	8.9023E-01	5.9753E-01	3.5600E-02	-7.0655E-04
15	-2.1980E-02	9.4364E-01	1.8623E-02	7.1451E-01	8.5030E-01	6.2332E-01	1.1273E-01	-1.5975E-03
16	-2.9202E-06	1.0000E 00	1.0002E-06	8.2628E-01	9.0900E-01	5.8550E-01	2.3061E-06	0.0
17	1.9762E-04	1.0004E 00	-3.5320E-04	8.2707E-01	9.0942E-01	5.8523E-01	-7.9412E-04	-2.0862E-06
18	-5.6112E-04	9.9645E-01	6.3852E-04	8.1924E-01	9.0525E-01	5.8790E-01	7.0965E-03	-1.8114E-04
19	-5.8763E-03	9.7935E-01	4.5363E-03	7.8533E-01	8.8725E-01	5.9945E-01	4.1297E-02	-7.0000E-04
20	-2.0274E-02	9.5876E-01	2.3509E-02	7.4450E-01	8.6606E-01	6.1312E-01	8.2476E-02	-1.5982E-03
21	-9.3156E-07	1.0000E 00	1.3258E-06	8.2628E-01	9.0900E-01	5.8550E-01	1.8852E-06	0.0
22	1.2817E-04	1.0004E 00	-2.0571E-04	8.2710E-01	9.0944E-01	5.8522E-01	-8.2600E-04	5.1260E-06
23	-5.2762E-04	9.9783E-01	7.7807E-04	8.2198E-01	9.0670E-01	5.8637E-01	4.3381E-03	-1.6177E-04
24	-5.7631E-03	9.8194E-01	7.6524E-03	7.9046E-01	8.8998E-01	5.9769E-01	3.6129E-02	-6.5345E-04
25	-1.6344E-02	9.7763E-01	2.5041E-02	7.8192E-01	8.8577E-01	6.0040E-01	4.4736E-02	-1.4260E-03
26	4.7074E-06	1.0000E 00	-4.6531E-07	8.2628E-01	9.0900E-01	5.8550E-01	-1.0166E-06	0.0
27	2.1688E-05	9.9987E-01	-4.2201E-04	8.2602E-01	9.0886E-01	5.8559E-01	2.6001E-04	-2.2292E-05
28	-6.3191E-04	9.9675E-01	1.7923E-03	8.1984E-01	9.0556E-01	5.8770E-01	6.4951E-03	-1.3822E-04
29	-9.5506E-03	9.8607E-01	1.0755E-02	7.9866E-01	8.9436E-01	5.9488E-01	2.7854E-02	-5.8061E-04
30	-1.3186E-02	9.8681E-01	2.4443E-02	8.0013E-01	8.9540E-01	5.9421E-01	2.6373E-02	-1.3283E-03
31	9.7898E-08	1.0000E 00	7.2461E-08	8.2628E-01	9.0900E-01	5.8550E-01	-3.0852E-08	0.0
32	-2.8920E-05	1.0000E 00	-8.5313E-07	8.2627E-01	9.0900E-01	5.8550E-01	6.0354E-06	-5.9605E-08
33	-5.0031E-04	9.9794E-01	2.3498E-03	8.2220E-01	9.0683E-01	5.8689E-01	4.1128E-03	-9.5606E-05
34	-3.6468E-03	9.9366E-01	1.0933E-02	8.1370E-01	9.0236E-01	5.8975E-01	1.2684E-02	-4.1550E-04
35	-8.8490E-03	9.9909E-01	2.0899E-02	8.2447E-01	9.0826E-01	5.8597E-01	1.8266E-03	-9.3639E-04
36	-5.4042E-05	1.0000E 00	1.4067E-06	8.2628E-01	9.0900E-01	5.8550E-01	-6.2505E-08	0.0
37	-2.1944E-03	9.9965E-01	1.0278E-02	8.2560E-01	9.0869E-01	5.8570E-01	6.5060E-04	-1.9957E-04
38	-7.2989E-03	1.0027E 00	1.8641E-02	8.3154E-01	9.1159E-01	5.8359E-01	-5.3012E-03	-9.7233E-04
39	1.9450E-05	1.0000E 00	-1.2770E-06	8.2623E-01	9.0900E-01	5.8550E-01	1.5430E-06	-2.3842E-07
40	-1.7207E-03	1.0002E 00	8.7813E-03	8.2660E-01	9.0921E-01	5.8536E-01	-3.2578E-04	-4.8351E-04
41	-5.3750E-03	1.0088E 00	1.4154E-02	8.4331E-01	9.1849E-01	5.7944E-01	-1.7674E-02	-8.4257E-04
42	6.3857E-05	1.0000E 00	-5.1033E-07	8.2628E-01	9.0900E-01	5.8550E-01	-2.8145E-06	0.0
43	-1.0743E-03	1.0048E 00	5.5395E-03	8.3570E-01	9.1405E-01	5.8227E-01	-9.5016E-03	-9.8228E-05
44	-3.4785E-03	1.0172E 00	8.2950E-03	8.6044E-01	9.2736E-01	5.7379E-01	-3.4448E-02	1.1021E-04
45	6.5382E-05	1.0000E 00	6.6105E-07	8.2628E-01	9.0900E-01	5.8550E-01	-1.4755E-06	0.0
46	-1.9495E-04	1.0062E 00	3.2946E-04	8.3848E-01	9.1552E-01	5.8133E-01	-1.2202E-02	3.1346E-04
47	-1.4345E-03	1.0191E 00	1.4609E-03	8.6421E-01	9.2935E-01	5.7253E-01	-3.8250E-02	3.6108E-04
48	-1.7213E-05	1.0000E 00	-3.0912E-06	8.2628E-01	9.0900E-01	5.8550E-01	2.5574E-06	0.0
49	1.1547E-03	1.0021E 00	-5.3668E-03	8.3049E-01	9.1126E-01	5.8405E-01	-4.2430E-03	4.7725E-04
50	1.1312E-03	1.0174E 00	-5.9264E-03	8.6084E-01	9.2756E-01	5.7367E-01	-3.4850E-02	3.9268E-04
51	-2.5664E-05	1.0000E 00	9.1044E-07	8.2628E-01	9.0900E-01	5.8550E-01	-3.6249E-06	-8.9407E-07
52	3.3019E-04	9.9812E-01	-2.5685E-03	8.2255E-01	9.0701E-01	5.8677E-01	3.7629E-03	1.4728E-04
53	-2.7245E-03	9.9569E-01	-8.5739E-03	8.1774E-01	9.0448E-01	5.8839E-01	8.6153E-03	6.2203E-04
54	4.2360E-03	1.0171E 00	-1.2258E-02	8.6024E-01	9.2729E-01	5.7383E-01	-3.4247E-02	3.0762E-04
55	-1.9960E-06	1.0000E 00	5.5398E-07	8.2626E-01	9.0900E-01	5.8550E-01	-7.4172E-07	0.0
56	-2.9422E-06	9.9984E-01	-8.0513E-05	8.2595E-01	9.0883E-01	5.8561E-01	3.1924E-04	3.9816E-05
57	4.9178E-04	9.9630E-01	-1.5547E-03	8.1906E-01	9.0515E-01	5.8796E-01	7.2775E-03	2.4289E-04
58	4.0032E-03	9.9318E-01	-8.3342E-03	8.1276E-01	9.0183E-01	5.9009E-01	1.3640E-02	7.3862E-04
59	8.7090E-03	1.0074E 00	-1.8842E-02	8.4090E-01	9.1701E-01	5.8033E-01	-1.4746E-02	1.3770E-03
60	6.3900E-08	1.0000E 00	2.9257E-08	8.2628E-01	9.0900E-01	5.8550E-01	1.7554E-08	0.0

61	2.2039E-06	1.0000E 00	-2.0130E-07	8.2629E-01	9.0900E-01	5.8550E-01	1.0152E-06	0.0
62	4.6065E-06	9.9995E-01	9.7510E-05	8.2619E-01	9.0895E-01	5.8553E-01	9.4665E-05	4.0531E-05
63	3.8105E-04	9.9538E-01	-7.0398E-04	8.2503E-01	9.0834E-01	5.8592E-01	1.2483E-03	3.2467E-04
64	4.6177E-03	9.8565E-01	-5.7945E-03	7.9783E-01	8.9387E-01	5.9519E-01	2.6656E-02	9.5767E-04
65	1.2426E-02	9.8492E-01	-2.0805E-02	7.9638E-01	8.9332E-01	5.9595E-01	3.0157E-02	1.5967E-03
66	-1.2242E-06	1.0000E 00	6.6648E-07	8.2628E-01	9.0900E-01	5.8550E-01	-9.5263E-07	0.0
67	-5.5731E-05	9.9996E-01	2.6841E-04	8.2619E-01	9.0895E-01	5.8553E-01	8.6848E-05	3.9101E-05
68	4.6522E-04	9.9817E-01	-3.5281E-04	8.2266E-01	9.0706E-01	5.8674E-01	3.6563E-03	3.7271E-04
69	4.8564E-03	9.8036E-01	-2.7549E-03	7.8733E-01	8.8830E-01	5.9877E-01	3.5281E-02	1.1017E-03
70	1.5947E-02	9.7253E-01	-2.2582E-02	7.7161E-01	8.8038E-01	6.0383E-01	5.4934E-02	2.2356E-03
71	5.2000E-07	1.0000E 00	1.1811E-06	8.2628E-01	9.0900E-01	5.8550E-01	-1.0371E-06	0.0
72	-1.7581E-04	1.0005E 00	4.4318E-04	8.2725E-01	9.0952E-01	5.8517E-01	-5.7786E-04	6.9459E-05
73	5.2308E-04	9.9738E-01	-8.2326E-04	8.2108E-01	9.0623E-01	5.8727E-01	5.2435E-03	4.3303E-04
74	4.3469E-03	9.6521E-01	-1.0326E-03	7.9699E-01	8.9341E-01	5.9549E-01	2.5549E-02	1.2249E-03
75	1.5301E-02	9.5685E-01	-2.0918E-02	7.4072E-01	8.6401E-01	6.1444E-01	8.6291E-02	2.4596E-03
76	-9.3170E-06	1.0000E 00	1.1401E-06	8.2629E-01	9.0901E-01	5.8550E-01	-6.2511E-06	0.0
77	-3.1364E-04	1.0008E 00	4.1099E-04	8.2785E-01	9.0984E-01	5.8496E-01	-1.5622E-03	1.1021E-04
78	6.1571E-04	9.9562E-01	-4.5323E-04	8.1759E-01	9.0436E-01	5.8846E-01	8.7646E-03	3.9458E-04
79	3.7387E-03	9.3787E-01	1.5427E-03	8.0222E-01	8.9619E-01	5.9371E-01	2.4267E-02	1.2546E-03
80	1.6589E-02	9.3584E-01	-7.0420E-03	6.9905E-01	8.4211E-01	6.2863E-01	1.2832E-01	2.2893E-03
81	1.4681E-05	1.0000E 00	-3.9287E-06	8.2627E-01	9.0900E-01	5.8550E-01	6.4345E-06	-2.3842E-07
82	-3.7110E-04	1.0015E 00	2.7750E-07	8.2918E-01	9.1055E-01	5.8451E-01	-2.9195E-03	1.3793E-04
83	6.5900E-04	9.9477E-01	6.2386E-05	8.1591E-01	9.0347E-01	5.8904E-01	1.0460E-02	3.4195E-04
84	3.6053E-03	9.9107E-01	2.2258E-04	8.0857E-01	8.9956E-01	5.9154E-01	1.7862E-02	1.4190E-03
85	1.5333E-02	9.3160E-01	4.9589E-04	6.9064E-01	8.3773E-01	6.3148E-01	1.3680E-01	2.2447E-03
86	-4.5056E-02	8.7147E-01	5.2880E-02	5.7141E-01	7.7848E-01	6.7004E-01	2.5705E-01	-2.1707E-03
87	-3.5988E-02	9.1032E-01	5.2210E-02	6.4858E-01	8.1760E-01	6.4456E-01	1.7922E-01	-2.2467E-03
88	-3.3784E-02	9.4264E-01	4.3679E-02	7.1254E-01	8.5012E-01	6.2343E-01	1.1471E-01	-2.1748E-03
89	-5.0901E-02	8.8796E-01	9.6221E-02	6.0410E-01	7.9665E-01	6.5650E-01	2.2408E-01	-2.3450E-03
90	-4.2421E-02	9.3413E-01	6.4558E-02	6.9560E-01	8.4262E-01	6.2830E-01	1.3173E-01	-2.3414E-03
91	-3.6743E-02	9.5134E-01	5.2829E-02	7.2579E-01	8.5958E-01	6.1731E-01	9.7318E-02	-2.2477E-03
92	-3.0295E-02	9.6688E-01	4.3055E-02	7.6061E-01	8.7520E-01	6.0721E-01	6.6232E-02	-2.1437E-03
93	-5.4242E-02	9.8562E-01	8.8754E-02	7.9776E-01	8.9802E-01	5.9253E-01	2.8767E-02	-2.3403E-03
94	-4.6070E-02	9.7015E-01	7.4103E-02	7.6709E-01	8.8056E-01	6.0375E-01	5.9700E-02	-2.3457E-03
95	-3.6281E-02	9.7745E-01	5.7150E-02	7.8155E-01	8.8659E-01	5.9962E-01	4.5109E-02	-2.3646E-03
96	-3.0882E-02	9.5423E-01	4.6382E-02	7.9502E-01	8.9351E-01	5.9542E-01	3.1530E-02	-2.2440E-03
97	-2.3698E-02	9.9107E-01	3.7286E-02	8.0857E-01	9.0030E-01	5.9107E-01	1.7858E-02	-1.9956E-03
98	-5.3065E-02	1.0379E 00	7.4583E-02	9.0143E-01	9.5242E-01	5.5794E-01	-7.5790E-02	-2.5518E-03
99	-4.6074E-02	1.0193E 00	6.6528E-02	8.6463E-01	9.3190E-01	5.7091E-01	-3.6677E-02	-2.4515E-03
100	-3.6719E-02	1.0069E 00	5.7865E-02	8.4003E-01	9.1612E-01	5.7967E-01	-1.3868E-02	-2.4751E-03
101	-3.1309E-02	1.0044E 00	4.7246E-02	8.3510E-01	9.1490E-01	5.8173E-01	-8.8563E-03	-2.4157E-03
102	-2.4037E-02	1.0034E 00	3.8620E-02	8.3304E-01	9.1340E-01	5.8269E-01	-6.8126E-03	-2.2246E-03
103	-1.7085E-02	1.0028E 00	3.0298E-02	8.3182E-01	9.1245E-01	5.8330E-01	-5.5856E-03	-1.8011E-03
104	-4.6446E-02	1.0793E 00	5.7982E-02	9.8360E-01	9.9648E-01	5.3045E-01	-1.5866E-01	-3.2176E-03
105	-4.3283E-02	1.0609E 00	5.6180E-02	9.4705E-01	9.7614E-01	5.4307E-01	-1.2180E-01	-3.0090E-03
106	-3.7439E-02	1.0447E 00	5.1583E-02	9.1488E-01	9.5824E-01	5.5427E-01	-8.9359E-02	-2.7945E-03
107	-3.1104E-02	1.0299E 00	4.5717E-02	8.8555E-01	9.4199E-01	5.6452E-01	-5.9775E-02	-2.6332E-03
108	-2.4552E-02	1.0215E 00	3.8194E-02	8.6898E-01	9.3269E-01	5.7041E-01	-4.3061E-02	-2.4490E-03
109	-1.6070E-02	1.0152E 00	3.1094E-02	8.5650E-01	9.2571E-01	5.7484E-01	-3.0475E-02	-2.1491E-03
110	-1.1982E-02	1.0094E 00	2.3783E-02	8.4496E-01	9.1930E-01	5.7893E-01	-1.8837E-02	-1.6087E-03
111	-4.0753E-02	1.1226E 00	4.0173E-02	1.0693E 00	1.0439E 00	5.0155E-01	-2.4511E-01	-3.0050E-03
112	-3.6543E-02	1.1002E 00	4.0705E-02	1.0250E 00	1.0188E 00	5.1676E-01	-2.0040E-01	-3.0499E-03
113	-3.2349E-02	1.0789E 00	4.0489E-02	9.8277E-01	9.9514E-01	5.3120E-01	-1.5783E-01	-3.0717E-03
114	-2.7068E-02	1.0588E 00	3.7412E-02	9.4295E-01	9.7298E-01	5.4504E-01	-1.1767E-01	-2.9265E-03
115	-2.1366E-02	1.0406E 00	3.2724E-02	9.0713E-01	9.5318E-01	5.5746E-01	-8.1535E-02	-2.7230E-03
116	-1.5581E-02	1.0276E 00	2.6595E-02	8.8101E-01	9.3880E-01	5.6653E-01	-5.5194E-02	-2.2206E-03
117	-1.0135E-02	1.0167E 00	2.0152E-02	8.5945E-01	9.2701E-01	5.7402E-01	-3.3455E-02	-1.4526E-03
118	-3.0311E-02	1.1553E 00	2.0571E-02	1.1352E 00	1.0812E 00	4.7934E-01	-3.1151E-01	-2.5444E-03
119	-2.6096E-02	1.1316E 00	2.4305E-02	1.0872E 00	1.0536E 00	4.9571E-01	-2.6318E-01	-2.4261E-03
120	-2.5143E-02	1.1056E 00	2.5852E-02	1.0357E 00	1.0243E 00	5.1338E-01	-2.1122E-01	-3.1004E-03
121	-2.1394E-02	1.0814E 00	2.7055E-02	9.6763E-01	9.9738E-01	5.2990E-01	-1.6272E-01	-3.8895E-03
122	-1.6559E-02	1.0601E 00	2.5074E-02	9.4551E-01	9.7399E-01	5.4441E-01	-1.2025E-01	-3.2305E-03
123	-1.2294E-02	1.0407E 00	2.0777E-02	9.0705E-01	9.5281E-01	5.5769E-01	-8.1455E-02	-1.9892E-03
124	-7.9003E-03	1.0243E 00	1.5377E-02	8.7438E-01	9.3497E-01	5.6896E-01	-4.8506E-02	-8.9663E-04
125	-1.7410E-02	1.1888E 00	-1.9812E-04	1.2006E 00	1.1195E 00	4.5711E-01	-3.7754E-01	-3.7336E-03
126	-1.7067E-02	1.1624E 00	6.5454E-03	1.1483E 00	1.0887E 00	4.7495E-01	-3.2476E-01	-4.0216E-03
127	-1.5916E-02	1.1399E 00	1.1852E-02	1.1038E 00	1.0629E 00	4.9017E-01	-2.7987E-01	-3.8633E-03
128	-1.4008E-02	1.1134E 00	1.4149E-02	1.0511E 00	1.0328E 00	5.0823E-01	-2.2676E-01	-4.5280E-03
129	-1.1412E-02	1.0313E 00	1.5414E-02	9.9248E-01	9.9984E-01	5.2838E-01	-1.6762E-01	-3.2635E-03
130	-8.4116E-03	1.0560E 00	1.3276E-02	9.3736E-01	9.6927E-01	5.4736E-01	-1.1203E-01	-1.3252E-04
131	-5.6753E-03	1.0306E 00	1.0003E-02	8.8701E-01	9.4174E-01	5.6467E-01	-6.1243E-02	3.4720E-04
132	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
134	-[illegible]E-03	1.1659E 00	-3.4045E-03	1.1553E 00	1.0928E 00	4.7257E-01	-3.3181E-01	-6.7139E-04
135	-4.5325E-03	1.1427E 00	1.6838E-03	1.1093E 00	1.0660E 00	4.8833E-01	-2.6539E-01	1.0203E-02
136	-4.0590E-03	1.1175E 00	3.0391E-03	1.0594E 00	1.0374E 00	5.0545E-01	-2.1506E-01	1.3699E-02
137	-3.6672E-03	1.0721E 00	1.6736E-03	9.6922E-01	9.8678E-01	5.3646E-01	-1.4416E-01	4.4667E-03
138	-3.1372E-03	1.0375E 00	3.2897E-03	9.0056E-01	9.4905E-01	5.6006E-01	-7.4910E-02	9.1606E-04
139	1.5925E-02	1.2542E 00	-4.5303E-02	1.3304E 00	1.1992E 00	4.1279E-01	-5.0843E-01	-2.2964E-03
140	1.2193E-02	1.2238E 00	-3.4599E-02	1.2701E 00	1.1618E 00	4.3325E-01	-4.4762E-01	-8.9264E-04
141	8.5600E-03	1.1941E 00	-2.2444E-02	1.2112E 00	1.1280E 00	4.5328E-01	-3.8810E-01	5.5561E-03
142	6.4878E-03	1.1428E 00	-1.4284E-02	1.1094E 00	1.0662E 00	4.8021E-01	-2.8558E-01	-5.4541E-03
143	3.82[illegible]8E-03	1.0857E 00	-1.6436E-02	1.0042E 00	1.0064E 00	5.2435E-01	-1.7541E-01	-2.9316E-03
144	1.2638E-03	1.0606E 00	-9.9225E-03	9.4654E-01	9.7428E-01	5.4423E-01	-1.2128E-01	-2.2788E-03
145	-3.6504E-04	1.0372E 00	-4.2389E-03	9.0006E-01	9.4879E-01	5.6022E-01	-7.4412E-02	4.1771E-04
146	3.3182E-02	1.1938E 00	-6.4747E-02	1.2105E 00	1.1274E 00	4.5257E-01	-3.8750E-01	5.4903E-03
147	2.7569E-02	1.1737E 00	-5.9497E-02	1.1707E 00	1.1036E 00	4.6629E-01	-3.4740E-01	3.8624E-03
148	2.1666E-02	1.1364E 00	-5.2051E-02	1.0967E 00	1.0601E 00	4.9182E-01	-2.7273E-01	-2.6913E-03
149	1.[illegible]635E-02	1.1017E 00	-4.2553E-02	1.0279E 00	1.0205E 00	5.1573E-01	-2.0330E-01	-2.2125E-03
150	1.0149E-02	1.0718E 00	-2.9285E-02	9.6869E-01	9.8692E-01	5.3637E-01	-1.4363E-01	-6.2799E-04
151	5.7493E-03	1.0499E 00	-1.8752E-02	9.2521E-01	9.6269E-01	5.5148E-01	-9.9769E-02	-1.3091E-03
152	2.3989E-03	1.0278E 00	-1.1147E-02	8.8139E-01	9.3871E-01	5.6660E-01	-5.5800E-02	4.3231E-04
153	4.3506E-02	1.0743E 00	-7.7469E-02	9.7369E-01	9.9233E-01	5.3302E-01	-1.4667E-01	3.6445E-03
154	3.6047E-02	1.0568E 00	-7.3249E-02	9.3897E-01	9.7284E-01	5.4513E-01	-1.1246E-01	3.5033E-03
155	2.8006E-02	1.0519E 00	-6.3210E-02	9.2925E-01	9.6679E-01	5.4891E-01	-1.0234E-01	3.1056E-03
156	2.6167E-02	1.0488E 00	-4.8663E-02	9.2297E-01	9.6251E-01	5.5159E-01	-9.7512E-02	1.9794E-03
157	1.3200E-02	1.0391E 00	-3.9550E-02	9.0377E-01	9.5142E-01	5.5857E-01	-7.8151E-02	1.0086E-03
158	7.4456E-03	1.0291E 00	-2.3065E-02	8.6394E-01	9.4030E-01	5.6558E-01	-5.8155E-02	3.8433E-04
159	4.6249E-02	9.9378E-01	-8.9458E-02	8.1395E-01	9.0667E-01	5.8659E-01	1.2440E-02	3.5203E-03
160	3.8018E-02	9.9590E-01	-7.4735E-02	8.1315E-01	9.0763E-01	5.8637E-01	8.1969E-03	3.5235E-03
161	2.9890E-02	9.9775E-01	-6.0448E-02	8.2181E-01	9.0855E-01	5.8579E-01	4.5070E-03	3.4735E-03
162	2.2146E-02	1.0078E 00	-4.7293E-02	8.4173E-01	9.1845E-01	5.7947E-01	-1.9578E-02	2.8223E-03
163	1.5035E-02	1.0158E 00	-3.5776E-02	8.5771E-01	9.2652E-01	5.7432E-01	-3.1654E-02	1.8958E-03
164	4.3253E-02	9.1188E-01	-9.8524E-02	6.5152E-01	8.2299E-01	6.4105E-01	1.7625E-01	3.6581E-03
165	3.5911E-02	9.3417E-01	-6.4869E-02	6.9574E-01	8.4268E-01	6.2826E-01	1.3166E-01	3.5809E-03
166	2.9302E-02	9.6164E-01	-5.2533E-02	7.5020E-01	8.7023E-01	6.1042E-01	7.6728E-02	3.3357E-03
167	2.2878E-02	9.8193E-01	-4.2710E-02	7.9044E-01	8.9092E-01	5.9709E-01	3.6147E-02	2.9164E-03
168	3.7678E-02	8.6579E-01	-5.1721E-02	5.6014E-01	7.7275E-01	6.7377E-01	2.6641E-01	3.0725E-03
169	3.2422E-02	9.2138E-01	-5.1231E-02	6.7037E-01	8.2673E-01	6.3733E-01	1.5724E-01	3.3816E-03
170	2.6842E-02	9.5401E-01	-4.2664E-02	7.3507E-01	8.6181E-01	6.1586E-01	5.1986E-02	3.1320E-03

B. CONVERGING-DIVERGING NOZZLE

Two test cases were run for the converging-diverging nozzle. The first case was for symmetric flow designed to yield sonic conditions in the throat of the nozzle. The second case dealt with supersonic flow in the diverging section by starting with the sonic line as the boundary of the nozzle mesh. Oswatitisch's two-dimensional nozzle [Ref. 9], $y = 1 + \sqrt{.2(x - 2.5)^2}$ with a semi-throat height of 1 at $x = 2.5$ was used in both cases. Boundaries for the subsonic nozzle were at $x = 0$ and $x = 5$.

1. Symmetric Solution

This problem was analyzed using the mesh shown in Fig. 10, which consists of 126 elements and 152 nodes. In the second card (the option card) IOPT(4) = 1 indicates that non-zero boundary velocities at the inlet and the exit will be read and applied by subroutine BNDRY. This option requires that the number of inlet and exit boundary nodes be specified in columns 36-40 of the next card. Perturbation velocities at the boundary nodes follow on the subsequent four cards. Subroutine BNDRY reads u and v respectively for the first boundary node and then continues reading u and v for each inlet and then each exit node in the order specified on the appropriate card.

2. Supersonic Case--Diverging Section

The mesh used for this case is sketched in Fig. 11 and input cards to STRANL-II follow on the next page. For this

case the options in effect are IOPT(4) = 1 and IOPT(5) = 1 which cause non-zero boundary velocities to be applied to the sonic line only.

INPUT TO STRANL-II

```
STEADY TRANSONIC FLOW CONVERGING DIVERGING NOZZLE-3 M=.375
          1
    1     .01         .375        1.0       16
 0.0         0.0        -8.978E-04-4.104E-02-3.641E-03-8.224E-02-8.377E-03-1.237E-01
-1.534E-02-1.656E-01-2.485E-02-2.078E-01-3.726E-02-2.503E-01-5.294E-02-2.926E-01
 0.0         0.0        -8.978E-04 4.104E-02-3.641E-03 8.224E-02-8.377E-03 1.237E-01
-1.534E-02 1.656E-01-2.485E-02 2.078E-01-3.726E-02 2.503E-01-5.294E-02 2.926E-01
  126   152    60    16    17    17
    2     1     9    10    10     9    17    18    18    17    25    26    26    25    33    34
   34    33    41    42    42    41    49    50    50    49    57    58    58    57    65    66
   66    65    73    74    74    73    81    82    82    81    89    90    90    89    97    98
   98    97   105   106   106   105   113   114   114   113   121   122   122   121   129   130
  130   129   137   138   138   137   145   146     3     2    10    11    11    10    18    19
   19    18    26    27    27    26    34    35    35    34    42    43    43    42    50    51
   51    50    58    59    59    58    66    67    67    66    74    75    75    74    82    83
   83    82    90    91    91    90    98    99    99    98   106   107   107   106   114   115
  115   114   122   123   123   122   130   131   131   130   138   139   139   138   146   147
    4     3    11    12    12    11    19    20    20    19    27    28    28    27    35    36
   36    35    43    44    44    43    51    52    52    51    59    60    60    59    67    68
   68    67    75    76    76    75    83    84    84    83    91    92    92    91    99   100
  100    99   107   108   108   107   115   116   116   115   123   124   124   123   131   132
  132   131   139   140   140   139   147   148     5     4    12    13    13    12    20    21
   21    20    28    29    29    28    36    37    37    36    44    45    45    44    52    53
   53    52    60    61    61    60    68    69    69    68    76    77    77    76    84    85
   85    84    92    93    93    92   100   101   101   100   108   109   109   108   116   117
  117   116   124   125   125   124   132   133   133   132   140   141   141   140   148   149
    6     5    13    14    14    13    21    22    22    21    29    30    30    29    37    38
   38    37    45    46    46    45    53    54    54    53    61    62    62    61    69    70
   70    69    77    78    78    77    85    86    86    85    93    94    94    93   101   102
  102   101   109   110   110   109   117   118   118   117   125   126   126   125   133   134
  134   133   141   142   142   141   149   150     7     6    14    15    15    14    22    23
   23    22    30    31    31    30    38    39    39    38    46    47    47    46    54    55
   55    54    62    63    63    62    70    71    71    70    78    79    79    78    86    87
   87    86    94    95    95    94   102   103   103   102   110   111   111   110   118   119
  119   118   126   127   127   126   134   135   135   134   142   143   143   142   150   151
    8     7    15    16    16    15    23    24    24    23    31    32    32    31    39    40
   40    39    47    48    48    47    55    56    56    55    63    64    64    63    71    72
   72    71    79    80    80    79    87    88    88    87    95    96    96    95   103   104
  104   103   111   112   112   111   119   120   120   119   127   128   128   127   135   136
  136   135   143   144   144   143   151   152
 0.0         0.0         0.0         2.143E-01 0.0         4.286E-01 0.0         6.429E-01
 0.0         8.571E-01 0.0         1.071E 00 0.0         1.286E 00 0.0         1.500E 00
 6.757E-01 0.0         6.757E-01 1.844E-01 6.757E-01 3.687E-01 6.757E-01 5.531E-01
 6.757E-01 7.375E-01 6.757E-01 9.218E-01 6.757E-01 1.106E 00 6.757E-01 1.291E 00
 1.183E 00 0.0         1.183E 00 1.658E-01 1.183E 00 3.316E-01 1.183E 00 4.974E-01
```

```
1.183E 00 6.632E-01 1.183E 00 8.290E-01 1.183E 00 9.949E-01 1.183E 00 1.161E 00
1.563E 00 0.0       1.563E 00 1.549E-01 1.563E 00 3.098E-01 1.563E 00 4.647E-01
1.563E 00 6.196E-01 1.563E 00 7.745E-01 1.563E 00 9.294E-01 1.563E 00 1.084E 00
1.848E 00 0.0       1.848E 00 1.488E-01 1.848E 00 2.976E-01 1.848E 00 4.464E-01
1.848E 00 5.952E-01 1.848E 00 7.441E-01 1.848E 00 8.929E-01 1.848E 00 1.042E 00
2.062E 00 0.0       2.062E 00 1.456E-01 2.062E 00 2.912E-01 2.062E 00 4.367E-01
2.062E 00 5.823E-01 2.062E 00 7.279E-01 2.062E 00 8.735E-01 2.062E 00 1.019E 00
2.222E 00 0.0       2.222E 00 1.440E-01 2.222E 00 2.879E-01 2.222E 00 4.319E-01
2.222E 00 5.758E-01 2.222E 00 7.198E-01 2.222E 00 8.637E-01 2.222E 00 1.008E 00
2.342E 00 0.0       2.342E 00 1.432E-01 2.342E 00 2.864E-01 2.342E 00 4.296E-01
2.342E 00 5.728E-01 2.342E 00 7.161E-01 2.342E 00 8.593E-01 2.342E 00 1.002E 00
2.432E 00 0.0       2.432E 00 1.429E-01 2.432E 00 2.858E-01 2.432E 00 4.288E-01
2.432E 00 5.717E-01 2.432E 00 7.146E-01 2.432E 00 8.575E-01 2.432E 00 1.000E 00
2.500E 00 0.0       2.500E 00 1.429E-01 2.500E 00 2.857E-01 2.500E 00 4.286E-01
2.500E 00 5.714E-01 2.500E 00 7.143E-01 2.500E 00 8.571E-01 2.500E 00 1.000E 00
2.568E 00 0.0       2.568E 00 1.429E-01 2.568E 00 2.858E-01 2.568E 00 4.288E-01
2.568E 00 5.717E-01 2.568E 00 7.146E-01 2.568E 00 8.575E-01 2.568E 00 1.000E 00
2.658E 00 0.0       2.658E 00 1.432E-01 2.658E 00 2.864E-01 2.658E 00 4.296E-01
2.658E 00 5.728E-01 2.658E 00 7.161E-01 2.658E 00 8.593E-01 2.658E 00 1.002E 00
2.778E 00 0.0       2.778E 00 1.440E-01 2.778E 00 2.879E-01 2.778E 00 4.319E-01
2.778E 00 5.758E-01 2.778E 00 7.198E-01 2.778E 00 8.637E-01 2.778E 00 1.008E 00
2.938E 00 0.0       2.938E 00 1.456E-01 2.938E 00 2.912E-01 2.938E 00 4.367E-01
2.938E 00 5.823E-01 2.938E 00 7.279E-01 2.938E 00 8.735E-01 2.938E 00 1.019E 00
3.152E 00 0.0       3.152E 00 1.488E-01 3.152E 00 2.976E-01 3.152E 00 4.464E-01
3.152E 00 5.952E-01 3.152E 00 7.441E-01 3.152E 00 8.929E-01 3.152E 00 1.042E 00
3.437E 00 0.0       3.437E 00 1.549E-01 3.437E 00 3.098E-01 3.437E 00 4.647E-01
3.437E 00 6.196E-01 3.437E 00 7.745E-01 3.437E 00 9.294E-01 3.437E 00 1.084E 00
3.817E 00 0.0       3.817E 00 1.658E-01 3.817E 00 3.316E-01 3.817E 00 4.974E-01
3.817E 00 6.632E-01 3.817E 00 8.290E-01 3.817E 00 9.949E-01 3.817E 00 1.161E 00
4.324E 00 0.0       4.324E 00 1.844E-01 4.324E 00 3.687E-01 4.324E 00 5.531E-01
4.324E 00 7.375E-01 4.324E 00 9.218E-01 4.324E 00 1.106E 00 4.324E 00 1.291E 00
5.000E 00 0.0       5.000E 00 2.143E-01 5.000E 00 4.286E-01 5.000E 00 6.429E-01
5.000E 00 8.571E-01 5.000E 00 1.071E 00 5.000E 00 1.286E 00 5.000E 00 1.500E 00
    1    2    3    4    5    6    7    8  145  146  147  148  149  150  151  152
    9   17   25   33   41   49   57   65   73   81   89   97  105  113  121  129
  137
   16   24   32   40   48   56   64   72   80   88   96  104  112  120  128  136
  144
-2.827E-01-2.270E-01-1.729E-01-1.252E-01-8.606E-02-5.520E-02-3.149E-02-1.352E-02
0.0       1.352E-02 3.149E-02 5.520E-02 8.606E-02 1.252E-01 1.729E-01 2.270E-01
2.827E-01
    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15   16
   17   18   19   20   21   22   23   24   25   26   27   28   29   30   31   32
   33   34   35   36   37   38   39   40   41   42   43   44   45   46   47   48
   49   50   51   52   53   54   55   56   57   58   59   60   61   62   63   64
   65   66   67   68   69   70   71   72   73   74   75   76   77   78   79   80
   81   82   83   84   85   86   87   88   89   90   91   92   93   94   95   96
   97   98   99  100  101  102  103  104  105  106  107  108  109  110  111  112
```

```
  113   114   115   116   117   118   119   120   121   122   123   124   125   126   127   128
  129   130   131   132   133   134   135   136   137   138   139   140   141   142   143   144
  145   146   147   148   149   150   151   152
STEADY TRANSONIC FLOW-CONVERGING DIVERGING NOZZLE M=.4388
 1     1
   10     .01        .4388       1.0       16
 0.0          0.0        -8.571E-04-3.995E-02-3.473E-03-8.004E-02-7.978E-03-1.204E-01
-1.458E-02-1.611E-01-2.356E-02-2.021E-01-3.525E-02-2.433E-01-4.997E-02-2.846E-01
 0.0          0.0        -8.571E-04 3.995E-02-3.473E-03 8.004E-02-7.978E-03 1.204E-01
-1.458E-02 1.611E-01-2.356E-02 2.021E-01-3.525E-02 2.433E-01-4.997E-02 2.846E-01
```

INPUT TO STRANL-II

```
STEADY SUPERSONIC FLOW ,SONIC LINE INPUT,  UNIFORM MESH
        1 1
   1      .01        1.0         1.0         8
 0.0        0.0        0.0         3.184E-03 0.0          5.544E-03 0.0          6.258E-03
 0.0        4.508E-03 0.0        -5.184E-04 0.0         -9.625E-03 0.0         -2.360E-02
  63    80     60     8      9      9
   2     1      9    10     10      9     17     18     18     17     25     26     26     25     33     34
  34    33     41    42     42     41     49     50     50     49     57     58     58     57     65     66
  66    65     73    74      3      2     10     11     11     10     18     19     19     18     26     27
  27    26     34    35     35     34     42     43     43     42     50     51     51     50     58     59
  59    58     66    67     67     66     74     75      4      3     11     12     12     11     19     20
  20    19     27    28     28     27     35     36     36     35     43     44     44     43     51     52
  52    51     59    60     60     59     67     68     68     67     75     76      5      4     12     13
  13    12     20    21     21     20     28     29     29     28     36     37     37     36     44     45
  45    44     52    53     53     52     60     61     61     60     68     69     69     68     76     77
   6     5     13    14     14     13     21     22     22     21     29     30     30     29     37     38
  38    37     45    46     46     45     53     54     54     53     61     62     62     61     69     70
  70    69     77    78      7      6     14     15     15     14     22     23     23     22     30     31
  31    30     38    39     39     38     46     47     47     46     54     55     55     54     62     63
  63    62     70    71     71     70     78     79      8      7     15     16     16     15     23     24
  24    23     31    32     32     31     39     40     40     39     47     48     48     47     55     56
  56    55     63    64     64     63     71     72     72     71     79     80
 1.155E-01 0.0         1.083E-01 1.438E-01 8.688E-02 2.875E-01 5.114E-02 4.310E-01
 1.110E-03 5.746E-01-6.322E-02 7.185E-01-1.418E-01 8.636E-01-2.348E-01 1.011E 00
 3.804E-01 0.0         3.741E-01 1.456E-01 3.550E-01 2.909E-01 3.232E-01 4.354E-01
 2.788E-01 5.790E-01 2.216E-01 7.217E-01 1.517E-01 8.638E-01 6.910E-02 1.006E 00
 6.454E-01 0.0         6.398E-01 1.494E-01 6.231E-01 2.982E-01 5.953E-01 4.459E-01
 5.564E-01 5.921E-01 5.064E-01 7.364E-01 4.452E-01 8.788E-01 3.730E-01 1.019E 00
 9.103E-01 0.0         9.055E-01 1.550E-01 8.913E-01 3.093E-01 8.674E-01 4.622E-01
 8.341E-01 6.132E-01 7.912E-01 7.618E-01 7.388E-01 9.077E-01 6.768E-01 1.051E 00
 1.175E 00 0.0         1.171E 00 1.622E-01 1.159E 00 3.236E-01 1.140E 00 4.837E-01
 1.112E 00 6.417E-01 1.076E 00 7.970E-01 1.032E 00 9.493E-01 9.807E-01 1.098E 00
 1.440E 00 0.0         1.437E 00 1.707E-01 1.427E 00 3.408E-01 1.412E 00 5.096E-01
 1.389E 00 6.764E-01 1.361E 00 8.408E-01 1.326E 00 1.002E 00 1.285E 00 1.160E 00
 1.705E 00 0.0         1.703E 00 1.805E-01 1.696E 00 3.605E-01 1.684E 00 5.394E-01
 1.667E 00 7.167E-01 1.646E 00 8.917E-01 1.619E 00 1.064E 00 1.588E 00 1.233E 00
 1.970E 00 0.0         1.969E 00 1.914E-01 1.964E 00 3.823E-01 1.956E 00 5.725E-01
 1.945E 00 7.615E-01 1.930E 00 9.488E-01 1.913E 00 1.134E 00 1.892E 00 1.317E 00
 2.235E 00 0.0         2.234E 00 2.031E-01 2.232E 00 4.059E-01 2.228E 00 6.083E-01
 2.222E 00 8.101E-01 2.215E 00 1.011E 00 2.206E 00 1.211E 00 2.196E 00 1.409E 00
 2.500E 00 0.0         2.500E 00 2.155E-01 2.500E 00 4.309E-01 2.500E 00 6.464E-01
 2.500E 00 8.619E-01 2.500E 00 1.077E 00 2.500E 00 1.293E 00 2.500E 00 1.508E 00
   1     2      3     4      5      6      7      8
   9    17     25    33     41     49     57     65     73
  16    24     32    40     48     56     64     72     80
```

```
1.374E-02 7.317E-02 1.289E-01 1.786E-01 2.215E-01 2.576E-01 2.873E-01 3.117E-01
3.333E-01
    1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16
   17     18     19     20     21     22     23     24     25     26     27     28     29     30     31     32
   33     34     35     36     37     38     39     40     41     42     43     44     45     46     47     48
   49     50     51     52     53     54     55     56     57     58     59     60     61     62     63     64
   65     66     67     68     69     70     71     72     73     74     75     76     77     78     79     80
```

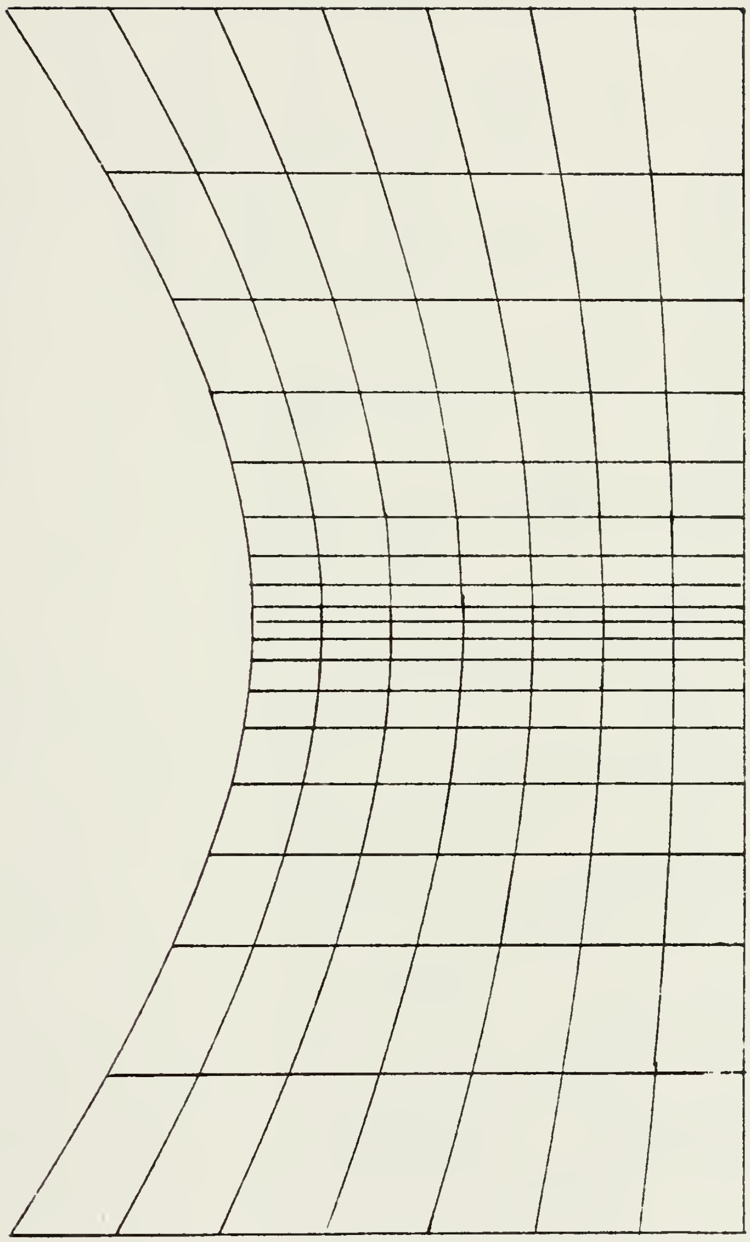


Figure 10 - Finite Element Mesh for the Converging-Diverging Nozzle.

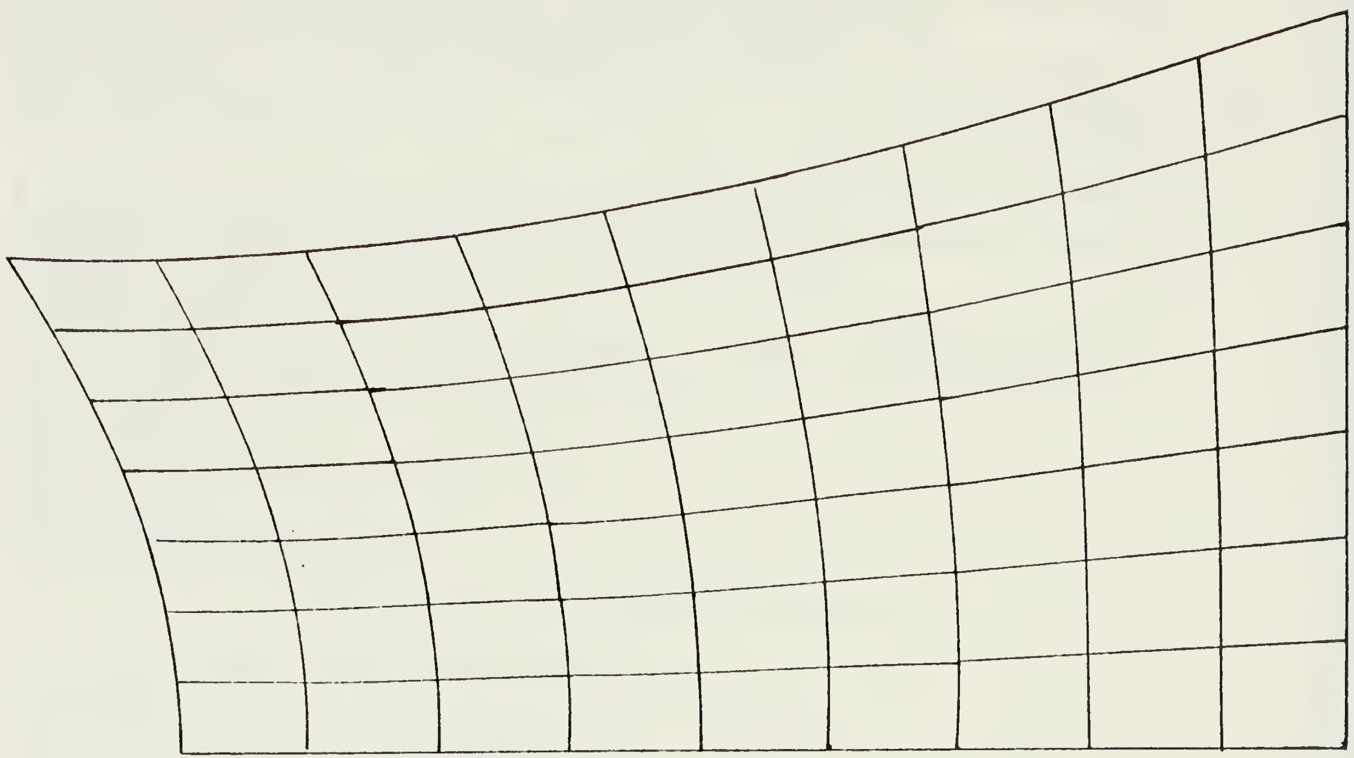


Figure 11 - Finite Element Mesh for the Supersonic Case

```
C     STEADY TRANSONIC FLOW ANALYSIS BY FINITE ELEMENT METHCD
C     USING LEAST SQUARES WITH TRIANGULAR AND CUACRALATERAL ELEMENTS
C     IOPT(1)=1 USE RESULTS OF PREVIOUS CASE AS STARTING SCLUTION
C     WHILE THE OTHER OPTICN IS IGNORED
C     IOPT(2)=1, READ IN NON-ZERO INITIAL GUESS
C     IOPT(3)=1, APPLY LINEARIZED BOUNDARY CONDITICN ON CHORDLINE
C     ICPT(4)=1, READ IN NON-ZERO BOUNDARY CONDITIONS
C     IOPT(5)=1, SONIC LINE VELOCITIES AS INPUTS
C     THE PROGRAM AS PRESENTLY DIMENSIONED ALLCWS THESE MAXIMA
C     200 ELEMENTS, 180 NODES, 50 NODES FOR EACH TYPE OF BCUNDARY CCND
C     MAX FULL BANDWIDTH = 84
C     DEVELOPED AND CODED BY STEVENS CHAN OF LOCKHEED-HUNTSVILLE ALA.
C
      DIMENSION TITLE(18),IOPT(20),NOD(200,4),S(540,84),SLP(540)
      DIMENSION WORK(540,42),UT(540,3),NDATA(3)
      DIMENSION X(180),Y(180),RML(180),RMLP(180),COF(180),NJNT(180)
      DIMENSION NIDS(3),NID(3,50),VAF(50),AR(100)
      DIMENSICN UB(50),VB(50)
      LOGICAL LR(50)
      EQUIVALENCE(NIDS(1),NFARF),(NIDS(2),NWAKE),(NIDS(3),NBODY)
      EQUIVALENCE (NDATA(1),NSK),(NDATA(2),NSM),(NDATA(3),NSC)
      DATA NDATA/9,13,14/
      DATA NEM,NPM,NCM,NA/200,180,84,50/
      NRM=3*NPM
      NHCM=NCM/2
      DATA PI/3.1415926/,GAMMA/1.40/
      IA=2*NA
      CCNST=0.5*(GAMMA-1.0)
      EXP=-GAMMA/(GAMMA-1.0)
C
C     READ TITLE, CONTROL KEYS AND PROGRAM PARAMETERS
C
      DATA NREAD,NWRITE,NPUNCH/10,8,3/
  100 READ(NREAD,805,END=2000)(TITLE(I),I=1,18)
  1C1 CONTINUE
      READ(NREAD,820) (IOPT(I),I=1,20)
      READ (NREAD,830) ITGIV,ZTEST,RMAC,F1,NFARF$
      WRITE (NWRITE,910) (TITLE(I),I=1,18),ZTEST
      WRITE (NWRITE,820) (IOPT(I),I=1,20)
      WRITE(6,999)ITGIV,ZTEST,RMAC,F1,NFARF$
  999 FORMAT(1H0,'ITGIV =',I3,5X,'ZTEST=',F8.4,5X,'RMAC=',F8.4,5X,
     1'F1=',F8.4,5X,'NFARF=',I5)
      IRES=0
      RFT=1.0
      SQMAC=RMAC**2
```

```
      IF (IOPT(4) .EQ. 1)READ(NREAD,840)(UB(I),VB(I),I=1,NFARF$)
      IF (IOPT(4) .EQ. 1) CALL BCCND(UB,VB,IOPT(5),NFARF$)
      IF (IOPT(1) .EQ. 1)GO TO 382
C
C     READ AND PRINT MESH DATA, BOUNDARY NODES, AND AIRFOIL SLOPE
C
      READ (NREAD,825) NELS,NPS,NBW,(NIDS(I),I=1,3)
      READ (NREAD,825) ((NOD(I,J),J=1,4),I=1,NELS)
      READ (NREAD,840) (X(I),Y(I),I=1,NPS)
      DO 110 I=1,3
      NS=NIDS(I)
  110 READ (NREAD,825) (NID(I,J),J=1,NS)
      READ (NREAD,840) (VAF(I),I=1,NBODY)
  120 READ (NREAD,825) (NJNT(I),I=1,NPS)
C
C     IF IOPT(3)=1,APPLY LINEARIZED BOUNDARY CONDITION ON CHORDLINE
C     OTHERWISE APPLY NONLINEAR BOUNDARY CONDITIONS ON AIRFOIL SURFACE
C
      IF (IOPT(3) .NE. 1)GO TO 116
      DO 115 J=1,NBODY
      I=NID(3,J)
  115 Y(I)=0.0
  116 CONTINUE
  200 WRITE (NWRITE,920) NELS,NPS,NBW
      WRITE (NWRITE,930)
      DO 220 N=1,NELS
  220 WRITE (NWRITE,825) N,(NOD(N,J),J=1,4)
      WRITE (NWRITE,935)
      DO 230 I=1,NPS
  230 WRITE (NWRITE,940) I, NJNT(I),X(I),Y(I)
      WRITE (NWRITE,951) (NID(1,I),I=1,NFARF)
      WRITE (NWRITE,952)(NID(2,I),I=1,NWAKE)
      WRITE (NWRITE,953) (NID(3,I),I=1,NBODY)
      WRITE (NWRITE,955) (VAF(I),I=1,NBODY)
C
C     REDEFINE MESH DATA, ECT. USING NEW NODAL NUMBERING SYSTEM
C
      DO 238 N=1,NELS
      DO 238 I=1,4
      IF (NOD(N,I) .EQ.0)GO TO 238
      KK=NOD(N,I)
      NOD(N,I)=NJNT(KK)
  238 CONTINUE
      DO 239 I=1,3
      IS=NIDS(I)
      DO 239 J=1,IS
      KK=NID(I,J)
  239 NID(I,J)=NJNT(KK)
```

```
      DO 243 I=1,NPS                                                    TRA00960
      RMLP(I)=X(I)                                                      TRA00970
  243 RML(I)=Y(I)                                                       TRA00980
      DO 244 II=1,NPS                                                   TRA00990
      I=NJNT(II)                                                        TRA01000
      X(I)=RMLP(II)                                                     TRA01010
  244 Y(I)=RML(II)                                                      TRA01020
C                                                                       TRA01030
C     HALF BANDWIDTH AND NUMBER OF EQUATIONS                            TRA01040
C                                                                       TRA01050
      NHBW=NBW/2                                                        TRA01060
      NEQ=3*NPS                                                         TRA01070
C                                                                       TRA01080
C     READ NONZERO INITIAL GUESS OR PROCEED WITH ZERO SOLUTION          TRA01090
C                                                                       TRA01100
      IF (IOPT(2) .NE. 1) GO TO 250                                     TRA01110
      READ (1,840) (S(I,NBW),I=1,NEQ)                                   TRA01120
      GO TO 295                                                         TRA01130
  250 DO 260 I=1,NEQ                                                    TRA01140
  260 SLP(I)=0.0                                                        TRA01150
C                                                                       TRA01160
C     ITERATIONS START HERE AND CHECKED IF ITGIV IS EXCEEDED. IF SO,    TRA01170
C     PRINT FAIL TO CONVERGE AND PROCEED TO NEXT CASE. OTHERWISE        TRA01180
C     CONTINUE TO ITERATE                                               TRA01190
C                                                                       TRA01200
  265 IRES=IRES+1                                                       TRA01210
    1 CONTINUE                                                          TRA01220
      IF (IRES .GT. ITGIV) GO TO 600                                    TRA01230
C     FORMULATE SYSTEM OF ALGEBRAIC EQUATIONS                           TRA01240
      DO 266 I=1,NEQ                                                    TRA01250
      DO 266 J=1,NBW                                                    TRA01260
  266 S(I,J)=0.0                                                        TRA01270
      NMAT=1                                                            TRA01280
      CALL NEWK(SQMAC,NRM,NCM,NEQ,NBW,NEM,NELS,NOC,SLP,S,CCF,NPM,       TRA01290
     1 X,Y,NMAT)                                                        TRA01300
C     IMPOSE B.C. FOR FARFIELD, LINE OF SYMMETRY, AND ON AIRFOIL        TRA01310
      IF (IOPT(4) .NE. 0) CALL BNDRY (S,UB,VB,VAF,NID,NFARF,NBODY,      TRA01320
     1 NWAKE,NBW,NHBW,NEQ,IOPT,IRES)                                    TRA01330
      IF(IOPT(4) .NE. 0) GO TO 276                                      TRA01340
      DO 274 I=1,NFARF                                                  TRA01350
      IE=3*NID(1,I)-3                                                   TRA01360
      DO 272 II=1,3                                                     TRA01370
      IE=IE+1                                                           TRA01380
      DO 270 K=1,NBW                                                    TRA01390
  270 S(IE,K)=0.0                                                       TRA01400
  272 S(IE,NHBW)=1.0                                                    TRA01410
  274 CONTINUE                                                          TRA01420
  276 CONTINUE                                                          TRA01430
```

```
      DO 280 I=1,NWAKE                                                  TRA01440
      IE=3*NID(2,I)                                                     TRA01450
      DO 278 K=1,NBW                                                    TRA01460
  278 S(IE,K)=0.0                                                       TRA01470
  280 S(IE,NHBW)=1.0                                                    TRA01480
      DO 285 J=1,NBODY                                                  TRA01490
      IE=3*NID(3,J)                                                     TRA01500
      DO 282 K=1,NBW                                                    TRA01510
  282 S(IE,K)=0.0                                                       TRA01520
      S(IE,NHBW)=1.0                                                    TRA01530
      IF (IOPT(3) .NE. 1) S(IE,NHBW-1)=-VAF(J)                          TRA01540
  285 S(IE,NBW)=VAF(J)                                                  TRA01550
C     CALL BANDED SOLVER TO SOLVE THE SYSTEM OF EQUATIONS               TRA01560
C                                                                       TRA01580
C     STORE STIFFNESS MATRIX AND INITIALIZE STARTING TIME DEPENDENT     TRA01590
C     UNKNOWNS                                                          TRA01600
C                                                                       TRA01610
      CALL STORE(S,WORK,NRM,NCM,NHCM,NHBW,NEQ,NSK)                      TRA01620
      IF  (IRES .GT. 1 .OR. RMAC .GT. .91) GO TO  401                   TRA01630
      CALL BNDEQ(S,NRM,NCM,NEQ,NHBW)                                    TRA01640
      DO 420 I=1,NRM                                                    TRA01650
      UT(I,3)=0.0                                                       TRA01660
      UT(I,2)=.5*S(I,NBW)                                               TRA01670
  420 UT(I,1)=S(I,NBW)                                                  TRA01680
      GO TO 295                                                         TRA01690
C     ASSEMBLE AND STORE THE DAMPING MATRIX                             TRA01700
  401 DO 400 I=1,NRM                                                    TRA01710
      DO 400 J=1,NCM                                                    TRA01720
  400 S(I,J)=0.0                                                        TRA01730
      NMAT=2                                                            TRA01740
      CALL NEWK(SQMAC,NRM,NCM,NEQ,NBW,NEM,NELS,NOD,SLP,S,CCF,NPM,       TRA01750
     1 X,Y,NMAT)                                                        TRA01760
      CALL STORE(S,WORK,NRM,NCM,NHCM,NHBW,NEQ,NSC)                      TRA01770
C     ASSEMBLE AND STORE THE MASS MATRIX                                TRA01780
      DO 410 I=1,NRM                                                    TRA01790
      DO 410 J=1,NCM                                                    TRA01800
  410 S(I,J)=0.0                                                        TRA01810
      NMAT=3                                                            TRA01820
      CALL NEWK(SQMAC,NRM,NCM,NEQ,NBW,NEM,NELS,NOD,SLP,S,CCF,NPM,       TRA01830
     1 X,Y,NMAT)                                                        TRA01840
      CALL STORE(S,WORK,NRM,NCM,NHCM,NHBW,NEQ,NSM)                      TRA01850
      CALL TIME(S,UT,WORK,NRM,NCM,NHCM,NHBW,NEQ)                        TRA01860
C                                                                       TRA01870
C     PRINT COMPUTED RESULTS                                            TRA01880
C                                                                       TRA01890
  295 WRITE (NWRITE,970) RMAC,RFT                                       TRA01900
      WRITE (NWRITE,975) IRES                                           TRA01910
      DO 305 I=1,NPS                                                    TRA01920
```

```
      J=NJNT(I)                                                         TRA01930
      II=3*NJNT(I)                                                      TRA01940
      PCT=S(II-2,NBW)                                                   TRA01950
      UPT=S(II-1,NBW)                                                   TRA01960
      V=S(II,NBW)                                                       TRA01970
      U=UPT+1.0                                                         TRA01980
      QSQ=U*U+V*V                                                       TRA01990
      ASQ=CONST*(1.0-QSQ)+1./SQMAC                                      TRA02000
      RML(J)=SQRT(QSQ/ASQ)                                              TRA02010
      PRATIO=(1.0+CONST*RML(J)**2)**EXP                                 TRA02020
      CP=-2.*UPT                                                        TRA02030
      COF(J)=SQMAC*(1.0+2.4*UPT)                                        TRA02040
      DELM=RML(J)-RMLP(J)                                               TRA02050
  305 WRITE (NWRITE,978) I,POT,U,V,COF(J),RML(J),PRATIO,CP,DELM         TRA02060
C                                                                       TRA02070
C     DISPLAY PRESSURE COEFFICIENT CP                                   TRA02080
C                                                                       TRA02090
      WRITE (NWRITE,985)                                                TRA02100
      ISTOP=0                                                           TRA02110
      IPNT=0                                                            TRA02120
      DO 320 J=1,NBODY                                                  TRA02130
      IF (J .EQ. NBODY) ISTOP =1                                        TRA02140
      I=NID(3,J)                                                        TRA02150
      UPT=S(3*I-1,NBW)                                                  TRA02160
      V=S(3*I,NBW)                                                      TRA02170
      CP=-2.*UPT                                                        TRA02180
  320 CONTINUE                                                          TRA02200
      IF (IRES .LT. 2) GO TO 382                                        TRA02210
C                                                                       TRA02220
C     CHECK CONVERGENCE. IF SO, PUNCH CONVERGED SOLUTION AND            TRA02230
C     PROCEED TO NEXT CASE. OTHERWISE, UPDATE SOLUTION AND CONTINUE     TRA02240
C     TO ITERATE                                                        TRA02250
C                                                                       TRA02260
      DO 340 I=1,NPS                                                    TRA02270
      PCTE=1.0-RMLP(I)/RML(I)                                           TRA02280
      IF (ABS(PCTE) .LT. ZTEST) GO TO 340                               TRA02290
      RFT=F1                                                            TRA02300
      GO TO 382                                                         TRA02310
  340 CONTINUE                                                          TRA02320
      WRITE (NPUNCH,840) (S(I,NBW),I=1,NEQ)                             TRA02330
      GO TO 100                                                         TRA02340
  382 DO 385 I=1,NPS                                                    TRA02350
  385 RMLP(I)=RML(I)                                                    TRA02360
      RFTC=1.0-RFT                                                      TRA02370
      DO 390 I=1,NEQ                                                    TRA02380
  390 SLP(I)=RFT*S(I,NBW)+RFTC*SLP(I)                                   TRA02390
      GO TO 265                                                         TRA02400
  600 WRITE (NWRITE,980)                                                TRA02410
```

```
      WRITE (1,840) (S(I,NBW),I=1,NEQ)                                  TRA02420
      GC TO 100                                                         TRA02430
  805 FCRMAT (18A4)                                                     TRA02440
  820 FCRMAT(40I2)                                                      TRA02450
  825 FCRMAT(16I5)                                                      TRA02460
  830 FCRMAT(I5,3F10.0,I5)                                              TRA02470
  840 FORMAT(1P8E10.3)                                                  TRA02480
  910 FCRMAT(1H1,2X,18A4//'  CONVERGENCE LIMIT =',F6.4//)               TRA02490
  920 FCRMAT(1H0,'NO. OF ELEMENTS=',I4,'  NO. CF NODES=',I4,            TRA02500
     1       '    FULL BANDWITH =',I4//)                                TRA02510
  930 FCRMAT ('0ELE. NO. AND ELEMENT NCDES'/)                           TRA02520
  935 FCRMAT ('0OLD NODE','  NEW NODE',6X,'X(I)','Y(I)'/)               TRA02530
  940 FORMAT (I6,I10,2E15.5)                                            TRA02540
  951 FORMAT ('0NODES AT FARFIELD'/(20I5))                              TRA02550
  952 FORMAT ('0NODES ON THE LINE OF SYMMETRY'/(20I5))                  TRA02560
  953 FCRMAT ('0NODES ON THE AIRFOIL'/(20I5))                           TRA02570
  955 FORMAT ('0SLOPE ALONG NODES ON AIRFOIL'/(8E15.5))                 TRA02580
  970 FORMAT (1H1,'MACH NUMBER=',F6.3,5X,'RELAX. FACTOR =',F6.4/)       TRA02590
  975 FCRMAT (1H0,4X,'NO. OF ITERATIONS =',I4/                          TRA02600
     1         1H0,7X,'NODE',8X,'PHIT',11X,'UCOM',11X,'VCCM',           TRA02610
     2       12X,'COF',11X,'LMAC',11X,'P/PO',13X,'CP',11X,'CELM'/)      TRA02620
  980 FCRMAT ('0FAIL TO CONVERGE IN SPECIFIED NO. OF ITERATIONS')       TRA02630
  978 FCRMAT (I10,1P8E15.4)                                             TRA02640
  985 FCRMAT (1H1)                                                      TRA02650
 2000 STOP                                                              TRA02660
      END                                                               TRA02700
```

SUBROUTINE BCOND

```
      SUBROUTINE BCOND(U,V,IOPT,N)                                          TRA00010
      DIMENSION U(N),V(N),UP(50),VP(50)                                     TRA00020
      DATA NWRITE/6/                                                        TRA00030
      DO 10 I=1,N                                                           TRA00040
      UP(I)=1+U(I)                                                          TRA00050
      VP(I)=V(I)                                                            TRA00060
   10 CONTINUE                                                              TRA00070
      IF (IOPT .EQ. 1) GO TO 150                                            TRA00080
      WRITE(NWRITE,100)                                                     TRA00090
  100 FORMAT ( ' UCOM AT INLET')                                            TRA00100
      N2=N/2                                                                TRA00110
      N3=N2+1                                                               TRA00120
      WRITE (NWRITE,110) (UP(I),I=1,N2)                                     TRA00130
      WRITE (NWRITE,120)                                                    TRA00140
  120 FORMAT ( ' VCOM AT INLET')                                            TRA00150
      WRITE (NWRITE,110) (VP(I),I=1,N2)                                     TRA00160
      WRITE (NWRITE,130)                                                    TRA00170
  130 FORMAT ( ' UCOM AT EXIT')                                             TRA00180
      WRITE (NWRITE,110) (UP(I),I=N3,N)                                     TRA00190
      WRITE (NWRITE,140)                                                    TRA00200
  140 FORMAT ( ' VCOM AT EXIT')                                             TRA00210
  110 FORMAT ( 1H ,8(F10.7,5X))                                             TRA00220
      WRITE (NWRITE,110) (VP(I),I=N3,N)                                     TRA00230
      GO TO 160                                                             TRA00240
  150 WRITE(NWRITE,121)                                                     TRA00250
      WRITE(NWRITE,110)(UP(I),I=1,N)                                        TRA00260
      WRITE(NWRITE,122)                                                     TRA00270
      WRITE(NWRITE,110)(VP(I),I=1,N)                                        TRA00280
  121 FORMAT( ' UCOM AT SONIC LINE')                                        TRA00290
  122 FORMAT( ' VCOM AT SONIC LINE')                                        TRA00300
  160 RETURN                                                                TRA00310
      END                                                                   TRA00320
```

SUBROUTINE BNDEQ

```
      SUBROUTINE BNDEQ(A,NRMAX,NCMAX,N,ITERM)                            TRA00010
 1201 CONTINUE                                                           TRA00020
C                                                                        TRA00030
C     EQUATION SOLVER FOR BANDED NON-SYMMENTRIC SYSTEM OF EQUATIONS      TRA00040
C     SOLUTION STORED IN THE LAST COLUMN AT (I,2*ITERM)                  TRA00050
C                                                                        TRA00060
      DIMENSION A(NRMAX,NCMAX)                                           TRA00070
      DATA NREAD,NWRITE,NPUNCH/4,8,3/                                    TRA00080
      CERO=1.E-6                                                         TRA00090
      PARE=CERO**2                                                       TRA00100
      NBND=2*ITERM                                                       TRA00110
      NBM=NBND-1                                                         TRA00120
C                                                                        TRA00130
C     BEGINS ELIMINATION OF THE LOWER LEFT                               TRA00140
C                                                                        TRA00150
      DO 1000 I=1,N                                                      TRA00160
      IF(ABS(A(I,ITERM)) .LT. CERO) GO TO 410                            TRA00170
      GO TO 430                                                          TRA00180
  410 IF(ABS(A(I,ITERM)) .LT. PARE) GO TO 1600                           TRA00190
      WRITE (6,420) A(I,ITERM), I                                        TRA00200
  420 FORMAT (' WARNING ILL-CONDITIONED A-MATRIX. A=',E16.6,' I=',I4)    TRA00210
  430 JLAST=MINO(I+ITERM-1,N)                                            TRA00220
      L=ITERM+1                                                          TRA00230
  431 CONTINUE                                                           TRA00240
      DO  500 J=I,JLAST                                                  TRA00250
      L=L-1                                                              TRA00260
      IF (ABS(A(J,L)) .LT. PARE) GO TO 500                               TRA00270
      B=A(J,L)                                                           TRA00280
      DO 450 K=L,NBND                                                    TRA00290
  450 A(J,K)=A(J,K)/B                                                    TRA00300
      IF(I .EQ. N) GO TO 1200                                            TRA00310
  500 CONTINUE                                                           TRA00320
      L=0                                                                TRA00330
      JFIRST=I+1                                                         TRA00340
      IF (JLAST .LE. I) GO TO 1000                                       TRA00350
      DO 900 J= JFIRST,JLAST                                             TRA00360
      L=L+1                                                              TRA00370
      IF ( ABS(A(J,ITERM-L)) .LT. PARE) GO TO 900                        TRA00380
      DO 600 K=ITERM,NBM                                                 TRA00390
  600 A(J,K-L) = A(J-L,K) -A(J,K-L)                                      TRA00400
      A(J,NBND) = A(J-L,NBND) - A(J,NBND)                                TRA00410
      IF (I .GE. N-ITERM+1) GO TO 900                                    TRA00420
      DO 800 K=1,L                                                       TRA00430
  800 A(J,NBND-K) = -A(J,NBND-K)                                         TRA00440
  900 CONTINUE                                                           TRA00450
 1000 CONTINUE                                                           TRA00460
```

```
C                                                                           TRA00470
C     BACK-SUBSTITUTION                                                     TRA00480
C                                                                           TRA00490
 120C L=ITERM -1                                                            TRA00500
      DO 1500 I=2,N                                                         TRA00510
      DO 1500 J=1,L                                                         TRA00520
      IF (N+1-I+J .GT. N) GO TO 1500                                        TRA00530
      ATEMP1=A(N+1-I,NBND)                                                  TRA00540
      ATEMP2=A(N+1-I+J,NBND)                                                TRA00550
      ATEMP3=A(N+1-I,ITERM+J)                                               TRA00560
      A(N+1-I,NBND)=A(N+1-I,NBND)-A(N+1-I+J,NBND)*A(N+1-I,ITERM+J)          TRA00570
 1500 CONTINUE                                                              TRA00580
      RETURN                                                                TRA00590
C                                                                           TRA00600
C     PRINT THE ENTIRE MATRIX IF ZERO ON MAIN DIAGONAL                      TRA00610
C                                                                           TRA00620
 1600 WRITE (6,1601)                                                        TRA00630
 1601 FORMAT (' COMPUTATION STOPED IN BNDEQ BECAUSE ZERO APPEARED ON        TRA00640
     1MAIN DIAGONAL. THE MATRIX FOLLOWS.')                                  TRA00650
      DO 1602 I=1,N                                                         TRA00660
 1602 WRITE (NWRITE,1603) (A(I,J), J=1,NBND)                                TRA00670
 1603 FORMAT (10E12.4)                                                      TRA00680
      STOP                                                                  TRA00690
      END                                                                   TRA00700
```

SUBROUTINE BNDRY

```
      SUBROUTINE BNDRY(S,UB,VB,VAF,NID,NFARF,NBODY,NWAKE,NBW            TRA00010
     1,NHBW,NEQ,IOPT,IRES)                                              TRA00020
      DIMENSION S(456,60),UB(50),VB(50),VAF(50),NID(3,50),IOPT(40)      TRA00030
      NBW$=NBW-1                                                        TRA00040
C     IF (IOPT(5) .EQ. 1 .AND. IRES .GT. 1) NFARF=NFARF/2               TRA00050
      DO 10 I=1,NFARF                                                   TRA00060
      IE=3*NID(1,I)-2                                                   TRA00070
      DO 10 J=1,2                                                       TRA00080
      IE =IE+1                                                          TRA00090
      NX=IE-NHBW                                                        TRA00100
      BC=UB(I)                                                          TRA00110
      IF (J.EQ.2) BC=VB(I)                                              TRA00120
      DO 41 L=1,NBW$                                                    TRA00130
      LX=NX+L                                                           TRA00140
      IF (LX .LE. 0 .OR.LX .GT. NEQ) GO TO 41                           TRA00150
   40 S(LX,NBW)=S(LX,NBW)-BC*S(LX,NBW-L)                                TRA00160
      S(LX,(NBW-L))=0.                                                  TRA00170
   41 CONTINUE                                                          TRA00180
      DO 30 K=1,NBW                                                     TRA00190
   31 S(IE,K)=0.0                                                       TRA00200
   30 CONTINUE                                                          TRA00210
      S(IE,NHBW)=1.0                                                    TRA00220
   10 S(IE,NBW)=BC                                                      TRA00230
      DO 100 I=1,NBODY                                                  TRA00240
      IE=3*NID(3,I)                                                     TRA00250
      BC=VAF(I)                                                         TRA00260
      NX=IE-NHBW                                                        TRA00270
      DO 410 L=1,NBW$                                                   TRA00280
      LX=NX+L                                                           TRA00290
      IF (LX .LE. 0 .OR. LX .GT.NEQ) GO TO 410                          TRA00300
  400 S(LX,NBW)=S(LX,NBW)-BC*S(LX,NBW-L)                                TRA00310
      S(LX,NBW-L)=0.0                                                   TRA00320
  410 CONTINUE                                                          TRA00330
      DO 300 K=1,NBW                                                    TRA00340
  300 S(IE,K)=0.0                                                       TRA00350
      S(IE,NHBW)=1.0                                                    TRA00360
  100 S(IE,NBW)=BC                                                      TRA00370
      DO 200 I=1,NWAKE                                                  TRA00380
      IE=3*NID(2,I)                                                     TRA00390
      DO 330 K=1,NBW                                                    TRA00400
  330 S(IE,K)=0.0                                                       TRA00410
  200 S(IE,NHBW)=1.                                                     TRA00420
      IF(IOPT(5) .EQ. 1 .AND. IRES .GT. 1)NFARF=NFARF*2                 TRA00430
      RETURN                                                            TRA00440
      END                                                               TRA00460
```

SUBROUTINE EMTC

```
      SUBROUTINE EMTC(A,AT,ATT,XL,YL,PEL,SQMAC)                          TRA00010
C                                                                        TRA00020
C     EVALUATE ELEMENT MATRIX FOR A TRIANGLE BY GAUSSIAN QUADRATURE      TRA00030
C     SUBROUTINE DERV CALLED TO EVALUATE SHAPE FUNCTION DERIVATIVES      TRA00040
C     AT THE GAUSSIAN POINTS                                             TRA00050
C                                                                        TRA00060
      DIMENSION A(9,9),P(9),Q(9),NP(5),B(3),C(3),XL(3),YL(3),S(3),       TRA00070
     1     DNX(9),DNXX(9),DNYY(9),PEL(9),                                TRA00080
     2     DN(9),AT(9,9),ATT(9,9)                                        TRA00090
      DIMENSION EINT(3,7),WT(7)                                          TRA00100
      DATA LMAX/7/,WT/0.225,3*0.13239415,3*.12593918/                    TRA00110
      DATA EINT/3*0.33333333,0.05961587,3*0.47014206,0.05961587,         TRA00120
     1   3*0.47014206,0.05961587,0.79742699,3*0.10128651,                TRA00130
     2   0.79742699,3*0.10128651,0.79742699/                             TRA00140
      DATA NP/1,2,3,1,2/,GAMMA/1.40/                                     TRA00150
      DATA NREAD,NWRITE,NPUNCH/4,8,3/                                    TRA00160
      DO 2 I=1,9                                                         TRA00170
      DO 2 J=1,9                                                         TRA00180
      AT(I,J)=0.                                                         TRA00190
      ATT(I,J)=0.                                                        TRA00200
    2 A(I,J)=0.0                                                         TRA00210
      DO 4 I=1,3                                                         TRA00220
      J=NP(I+1)                                                          TRA00230
      K=NP(I+2)                                                          TRA00240
      B(I)=YL(J)-YL(K)                                                   TRA00250
    4 C(I)=XL(K)-XL(J)                                                   TRA00260
      AREA=0.5*(B(2)*C(3)-B(3)*C(2))                                     TRA00270
      CST1=1.0-SQMAC                                                     TRA00280
      CST2=SQMAC*(1.0+GAMMA)                                             TRA00290
      DO 100 L=1,LMAX                                                    TRA00300
      DO 10 I=1,3                                                        TRA00310
   10 S(I)=EINT(I,L)                                                     TRA00320
      CALL DERV(AREA,B,C,S,DN,DNX,DNXX,DNYY)                             TRA00330
      U=0.0                                                              TRA00340
      UX=0.0                                                             TRA00350
      DO 30 I=1,9                                                        TRA00360
      U=U+DNX(I)*PEL(I)                                                  TRA00370
   30 UX=UX+DNXX(I)*PEL(I)                                               TRA00380
      ALPHA=CST1-CST2*U                                                  TRA00390
      DO 40 I=1,9                                                        TRA00400
      P(I)=ALPHA*DNXX(I)+DNYY(I)                                         TRA00410
   40 Q(I)=P(I)-CST2*UX*DNX(I)                                           TRA00420
      WEIGHT=WT(L)*AREA                                                  TRA00430
      DO 60 I=1,9                                                        TRA00440
      CST=WEIGHT*Q(I)                                                    TRA00450
```

```
      CC 60 J=1,9                                                       TRA00460
      AT(I,J)=AT(I,J)-2*CST*SQMAC*DNX(J)                                TRA00470
      ATT(I,J)=ATT(I,J)-CST*SQMAC*DN(J)                                 TRA00480
   60 A(I,J)=A(I,J)+CST*P(J)                                            TRA00490
  1CC CCNTINUE                                                          TRA00500
      RETURN                                                            TRA00510
      END                                                               TRA00530
```

SUBROUTINE EMQT

```
      SLBROUTINE EMQT(XQ,YQ,PMQ,SQMAC,EG,EQT,EQTT,NTRS)
C
C     GENERATE MATRIX FOR A QUADRILATERAL OR TRIANGLE
C     SLBROUTINE EMTC CALLED TO GENERATE MATRIX FCR A BASIC TRIANGLE
C
      CIMENSION EQ(12,12),ET(9,9),XQ(4),YQ(4),XT(3),YT(3),MP(3,4)
      CIMENSION PMQ(12),PMT(9),EQT(12,12),EQTT(12,12),ETT(9,9),
     1     ETTT(9,9)
      CATA MP/1,2,3,3,4,1,2,3,4,4,1,2/
      CATA NREAD,NWRITE,NPUNCH/4,8,3/
      FTCR=1.0
      IF (NTRS .EQ. 4) FTOR=.5
      CO 100 I=1,12
      CC 100 J=1,12
      EGT(I,J)=0.0
      EGTT(I,J)=0.0
  10C EQ(I,J)=0.0
      DO 150 II=1,NTRS
      CC 105 I=1,3
      NI=MP(I,II)
      IT=3*(I-1)
      IG=3*(NI-1)
      CO 102 J=1,3
      IT=IT+1
      IG=IQ+1
  102 PMT(IT)=PMQ(IQ)
      XT(I)=XQ(NI)
  1C5 YT(I)=YQ(NI)
      CALL EMTC(ET,ETT,ETTT,XT,YT,PMT,SQMAC)
      CC 130 K=1,3
      NR=3*(MP(K,II)-1)
      IE=3*(K-1)
      CC 130 KK=1,3
      NR=NR+1
      IE=IE+1
      CC 130 L=1,3
      NC=3*(MP(L,II)-1)
      JE=3*(L-1)
      CO 130 LL=1,3
      NC=NC+1
      JE=JE+1
      EQT(NR,NC)=EQT(NR,NC)+ETT(IE,JE)*FTOR
      EQTT(NR,NC)=EQTT(NR,NC)+ETTT(IE,JE)*FTOR
  130 EG(NR,NC)=EQ(NR,NC)+ET(IE,JE)*FTCR
  15C CCNTINUE
```

```
      RETURN                                                         TRAC0460
      END                                                            TRA00480
```

SUBROUTINE DERV

```
      SUBROUTINE DERV(AREA,B,C,S,DN,DNX,DNXX,DNYY)                      TRA00010
C                                                                       TRA00020
C     EVALUATE SHAPE FUNCTION DERIVATIVES AT GAUSIAN POINT              TRA00030
C                                                                       TRA00040
      DIMENSION B(3),C(3),S(3),DN(9),DNX(9),DNXX(9),DNYY(9),NP(5)       TRA00050
      DATA NP/1,2,3,1,2/                                                TRA00060
      DATA NREAD,NWRITE,NPUNCH/4,8,3/                                   TRA00070
      TWOA=2.*AREA                                                      TRA00080
      TWOASQ=TWOA**2                                                    TRA00090
      DO 200 I=1,3                                                      TRA00100
      J=NP(I+1)                                                         TRA00110
      K=NP(I+2)                                                         TRA00120
      SI=S(I)                                                           TRA00130
      SJ=S(J)                                                           TRA00140
      SK=S(K)                                                           TRA00150
      BI=B(I)                                                           TRA00160
      BJ=B(J)                                                           TRA00170
      BK=B(K)                                                           TRA00180
      CI=C(I)                                                           TRA00190
      CJ=C(J)                                                           TRA00200
      CK=C(K)                                                           TRA00210
      SISQ=SI*SI                                                        TRA00220
      BISQ=BI*BI                                                        TRA00230
      CISQ=CI*CI                                                        TRA00240
      ALFA=0.5*(CK-CJ)                                                  TRA00250
      BETA=0.5*(BJ-BK)                                                  TRA00260
      H=SI*SJ*SK                                                        TRA00270
      HX=BI*SJ*SK+BJ*SK*SI+BK*SI*SJ                                     TRA00280
      HXX=2.*(SI*BJ*BK+SJ*BK*BI+SK*BI*BJ)                               TRA00290
      HYY=2.*(SI*CJ*CK+SJ*CK*CI+SK*CI*CJ)                               TRA00300
      CSS=6.*SI*(1.0-SI)                                                TRA00310
      CS=6.*(1.0-2.0*SI)                                                TRA00320
      L=3*I-2                                                           TRA00330
      DN(L)=SI*SI*(3-2*SI)+2*H                                          TRA00340
      DNX(L)=BI*CSS+2.*HX                                               TRA00350
      DNXX(L)=BISQ*CS+2.*HXX                                            TRA00360
      DNYY(L)=CISQ*CS+2.*HYY                                            TRA00370
      CS=CK*SJ-CJ*SK                                                    TRA00380
      L=L+1                                                             TRA00390
      DN(L)=SISQ*CS+ALPHA*H                                             TRA00400
      DNX(L)=2.*BI*SI*CS+TWOA*SISQ+ALFA*HX                              TRA00410
      DNXX(L)=2.*BISQ*CS+4.*BI*TWOA*SI+ALFA*HXX                         TRA00420
      DNYY(L)=2.*CISQ*CS+ALFA*HYY                                       TRA00430
      BS=BJ*SK-BK*SJ                                                    TRA00440
      L=L+1                                                             TRA00450
```

```
      DN(L)=SISQ*BS+BETA*H
      DNX(L)=2.*BI*SI*BS+BETA*HX
      DNXX(L)=2.*BISQ*BS+BETA*HXX
  200 DNYY(L)=2.*CISQ*BS+4.*CI*TWOA*SI+BETA*HYY
      DO 300 I=1,9
      DNX(I)=DNX(I)/TWOA
      DNXX(I)=DNXX(I)/TWOASQ
  300 DNYY(I)=DNYY(I)/TWOASQ
      RETURN
      END
```

SUBROUTINE NEWK

```
      SUBROUTINE NEWK(SQMAC,NRM,NCM,NEQ,NBW,NEM,NELS,NOD,SLP,S,         TRA00010
     1   COEF,NPM,X,Y,NMAT)                                             TRA00020
C                                                                       TRA00030
C     GENERATE SYSTEM MATRIX BY ASSEMBELLING CONTRIBUTIONS FROM         TRA00040
C     ALL THE ELEMENTS                                                  TRA00050
C     SUBROUTINE EMQT CALLED TO GENERATE ELEMENT MATRIX                 TRA00060
C                                                                       TRA00070
      DIMENSION COEF(NPM),X(NPM),Y(NPM),XQ(4),YQ(4),PM(12),BB(12,12)    TRA00080
      DIMENSION BBC(12,12),BBM(12,12)                                   TRA00090
      DIMENSION NOD(NEM,4),S(NRM,NCM),SLP(NRM)                          TRA00100
      DATA NREAD,NWRITE,NPUNCH/4,8,3/                                   TRA00110
      NHBW=NBW/2                                                        TRA00120
      DO 480 N=1,NELS                                                   TRA00130
      I1=1                                                              TRA00140
      IF (NOD(N,4)) 402,402,404                                         TRA00150
  402 NPEL=3                                                            TRA00160
      NTRS=1                                                            TRA00170
      GO TO 410                                                         TRA00180
  404 NPEL=4                                                            TRA00190
      NTRS=4                                                            TRA00200
      I2=0                                                              TRA00210
      DO 408 I=1,4                                                      TRA00220
      NI=NOD(N,I)                                                       TRA00230
  408 IF (COEF(NI) .GT. 1.00)  I2=I2+1                                  TRA00240
      IF(I2.EQ.0) NTRS=2                                                TRA00250
      IF(I2 .EQ. 4) I1=3                                                TRA00260
  410 DO 425 I=1,NPEL                                                   TRA00270
      NI=NOD(N,I)                                                       TRA00280
      XQ(I)=X(NI)                                                       TRA00290
      YQ(I)=Y(NI)                                                       TRA00300
      DO 425 J=1,3                                                      TRA00310
      IS=3*(NI-1)+J                                                     TRA00320
      IE=3*(I-1)+J                                                      TRA00330
  425 PM(IE)=SLP(IS)                                                    TRA00340
      CALL EMQT(XQ,YQ,PM,SQMAC,BB,BBC,BBM,NTRS)                         TRA00350
      DO 450 I=I1,NPEL                                                  TRA00360
      NR=3*(NOD(N,I)-1)                                                 TRA00370
      IE=3*(I-1)                                                        TRA00380
      DO 450 II=1,3                                                     TRA00390
      NR=NR+1                                                           TRA00400
      IE=IE+1                                                           TRA00410
      DO 450 J=1,NPEL                                                   TRA00420
      NC=3*(NOD(N,J)-1)-NR+NHBW                                         TRA00430
      JE=3*(J-1)                                                        TRA00440
      DO 450 JJ=1,3                                                     TRA00450
```

```
      NC=NC+1                                                           TRA00460
      JE=JE+1                                                           TRA00470
      IF (NMAT-2) 443,442,441                                           TRA00480
  441 S(NR,NC)=S(NR,NC)+BBM(IE,JE)                                      TRA00490
      GO TO 450                                                         TRA00500
  442 S(NR,NC)=S(NR,NC)+BBC(IE,JE)                                      TRA00510
      GO TO 450                                                         TRA00520
  443 S(NR,NC)=S(NR,NC)+BB(IE,JE)                                       TRA00530
  450 CONTINUE                                                          TRA00540
  480 CONTINUE                                                          TRA00550
      RETURN                                                            TRA00560
      END                                                               TRA00580
```

SUBROUTINE STORE

```
      SUBROUTINE STORE (S,WORK,NRM,NCM,NHCM,NHBW,NEQ,NDATA)            STO00010
      DIMENSION S(NRM,NCM),WORK(NRM,NHCM)                              STO00020
      REWIND NDATA                                                     STO00030
      DO 10 I=1,NRM                                                    STO00040
      DO 10 J=1,NHCM                                                   STO00050
   10 WORK(I,J)=0.0                                                    STO00060
      DO 20 I=1,NEQ                                                    STO00070
      DO 20 J=1,NHBW                                                   STO00080
   20 WORK(I,J)=S(I,J)                                                 STO00090
      WRITE (NDATA) WORK                                               STO00100
      DO 30 I=1,NEQ                                                    STO00110
      DO 30 J=1,NHBW                                                   STO00120
      JA=NHBW+J                                                        STO00130
   30 WORK(I,J)=S(I,JA)                                                STO00140
      WRITE (NDATA) WORK                                               STO00150
      REWIND NDATA                                                     STO00160
      RETURN                                                           STO00170
      END                                                              STO00180
```

SUBROUTINE TIME

```
      SUBROUTINE TIME(S,U,WORK,NRM,NCM,NHCM,NHBW,NEQ)
C
C     HCULBOLT INTEGRATION FOR UNSTEADY PROBLEMS
C
      DIMENSION S(NRM,NCM),WORK(NRM,NHCM),U(NRM,3),NDATA(3)
      CATA NDATA/9,13,14/
      CATA DELT/100./
      NBW=2*NHBW
      NBW1=NBW-1
      AC=2./DELT**2
      A1=11./(6*DELT)
      A2=5./DELT**2
      A3=3./DELT
      A4=-2.*A0
      A5=-A3/2.
      A6=A0/2.
      A7=A3/9.
      CC 100 I=1,NRM
      DO 100 J=1,NCM
  100 S(I,J)=0.0
C FCRM LOWER TRIANGULAR PLUS DIAGCNAL EFFECTIVE MATRIX
      DO 25 K=1,3
      NCATA$=NDATA(K)
      REAC (NCATA$) WORK
      IF (K-2)1,2,3
    1 CCNST=1
      GC TO 4
    2 CCNST=AC
      GC TO 4
    3 CCNST=A1
    4 DO 10 J=1,NHBW
      CC 10 I=1,NEQ
   10 S(I,J)=WORK(I,J)*CONST+S(I,J)
C     MULTIPLY CONTRIBUTING MATRICES BY THE TIME CEPENDENT
C     VECTORS TO FORM RIGHT HAND SIDE
      DO 20 J=1,NHBW
      JM=NHBW+1-J
      IM=0
      CC 20 I=JM,NEQ
      IM=IM+1
      IF (K-2) 11,12,13
   11 UM=0.
      GC TO 20
   12 UM=A2*U(IM,1)+A4*U(IM,2)+A6*U(IM,3)
      GC TO 20
```

```
13  UM=A3*U(IM,1)+A5*U(IM,2)+A7*U(IM,3)                          TIM00460
20  S(IM,NBW)=S(IM,NBW)+WORK(I,J)*UM                               TIM00470
25  CCNTINUE                                                       TIMC0480
C      FCRM UPPER TRIANGULAR EFFECTIVE MATRIX                      TIM00490
    CC 45 K=1,3                                                    TIM00500
    NCATA$=NDATA(K)                                                TIM00510
    READ (NCATA$) WORK                                             TIM00520
    IF(K-2) 21,22,23                                               TIM00530
21  CCNST=1                                                        TIM00540
    GO TO 24                                                       TIM00550
22  CCNST=A0                                                       TIM00560
    GC TO 24                                                       TIM00570
23  CCNST=A1                                                       TIM00580
24  DO 30 J=1,NHBW                                                 TIM00590
    DO 30 I=1,NEQ                                                  TIM00600
    JA=J+NHBW                                                      TIM00610
3C  S(I,JA)=S(I,JA)+WORK(I,J)*CONST                                TIM00620
    MAXJ=NHBW-1                                                    TIM00630
    CC 40 J=1,MAXJ                                                 TIM00640
    JA=NHBW+J                                                      TIM00650
    MAXI=NEQ-J                                                     TIM00660
    IM=J                                                           TIM00670
    CC 40 I=1,MAXI                                                 TIM00680
    IM=IM+1                                                        TIM00690
    IF (K-2) 31,32,33                                              TIM00700
31  UM=0                                                           TIM00710
    GC TO 40                                                       TIM00720
32  UM=A2*U(IM,1)+A4*U(IM,2)+A6*U(IM,3)                            TIM00730
    GC TO 40                                                       TIM00740
33  UM=A3*U(IM,1)+A5*U(IM,2)+A7*U(IM,3)                            TIM00750
40  S(IM,NBW)=S(IM,NBW)+WORK(I,J)*UM                               TIM00760
45  CCNTINUE                                                       TIM00770
843 FCRMAT(1H ,4(F10.5,5X))                                        TIM00780
    CALL BNDEQ(S,NRM,NCM,NEQ,NHBW)                                 TIM00790
    CC 50 I=1,NRM                                                  TIM00800
    U(I,3)=U(I,2)                                                  TIM00810
    U(I,2)=U(I,1)                                                  TIM00820
50  U(I,1)=S(I,NBW)                                                TIM00830
8CC FORMAT(1H ,6(F10.5,2X))                                        TIM00840
    RETURN                                                         TIM00850
    END                                                            TIM00870
```

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